

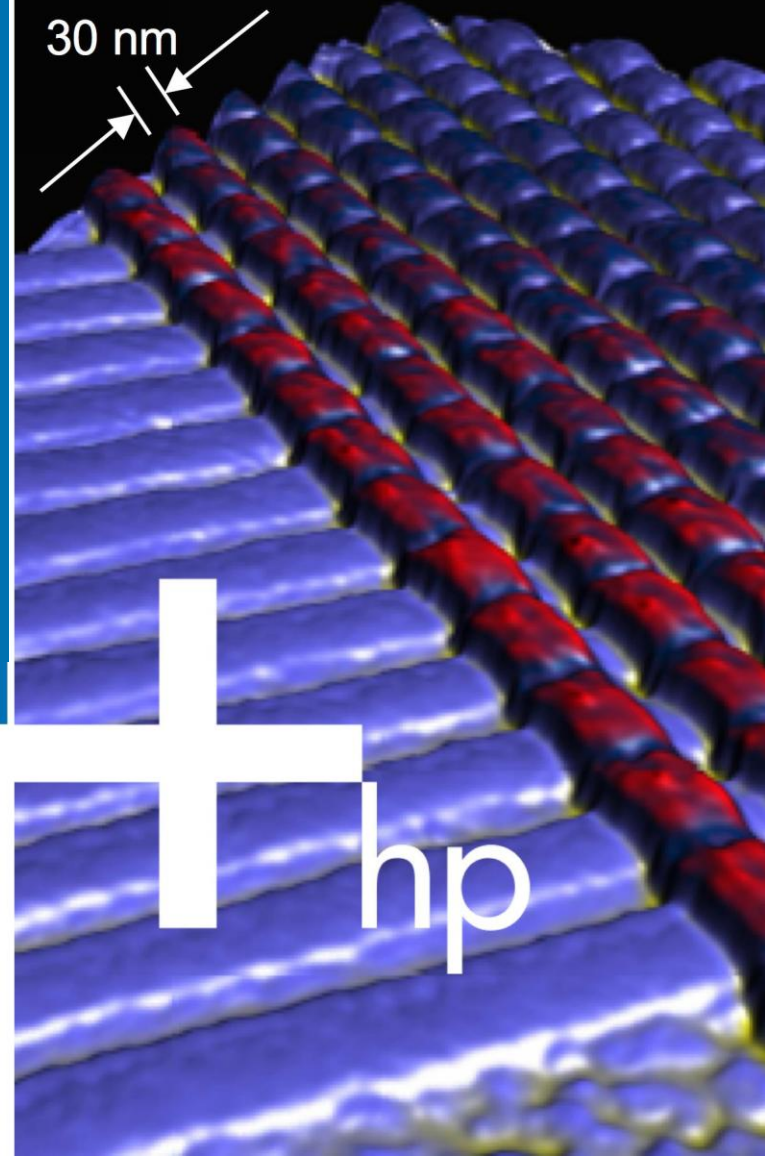


Stable learning in networks of unreliable, memristive nanodevices

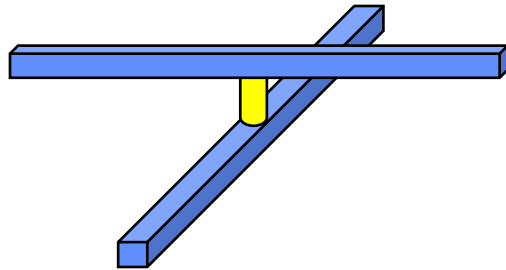
Adaptive Massively Parallel Computing Workshop

March 2-3, 2009
Portland, OR, USA

Greg Snider
IQSL
Hewlett-Packard Laboratories



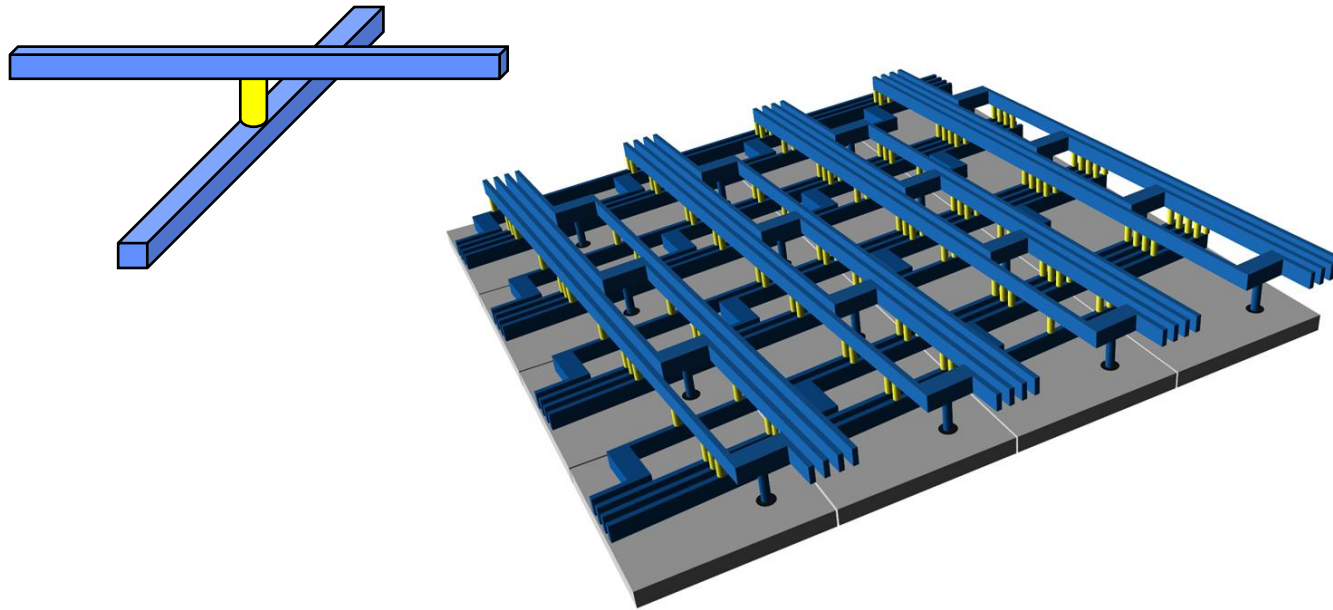
Outline



1. Memristive nanodevices

(synapses)

Outline



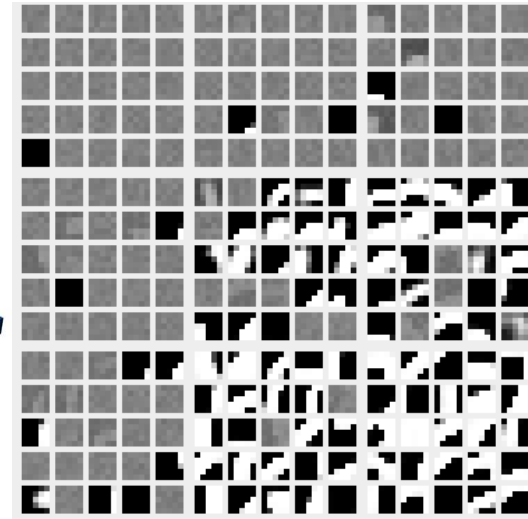
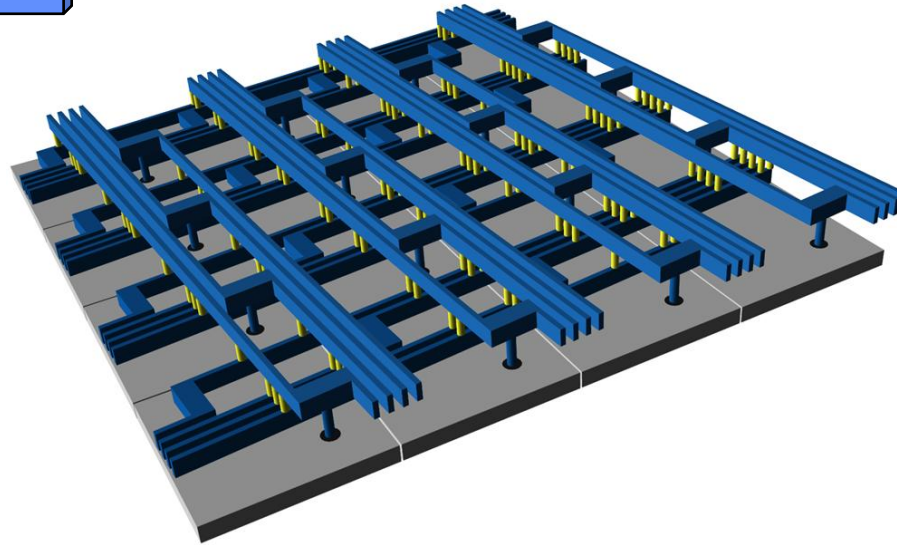
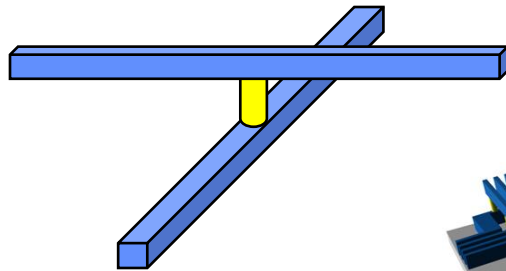
**1. Memristive
nanodevices**

(synapses)

**2. Adaptive resonance
networks**

(nano + CMOS)

Outline



1. Memristive nanodevices

(synapses)

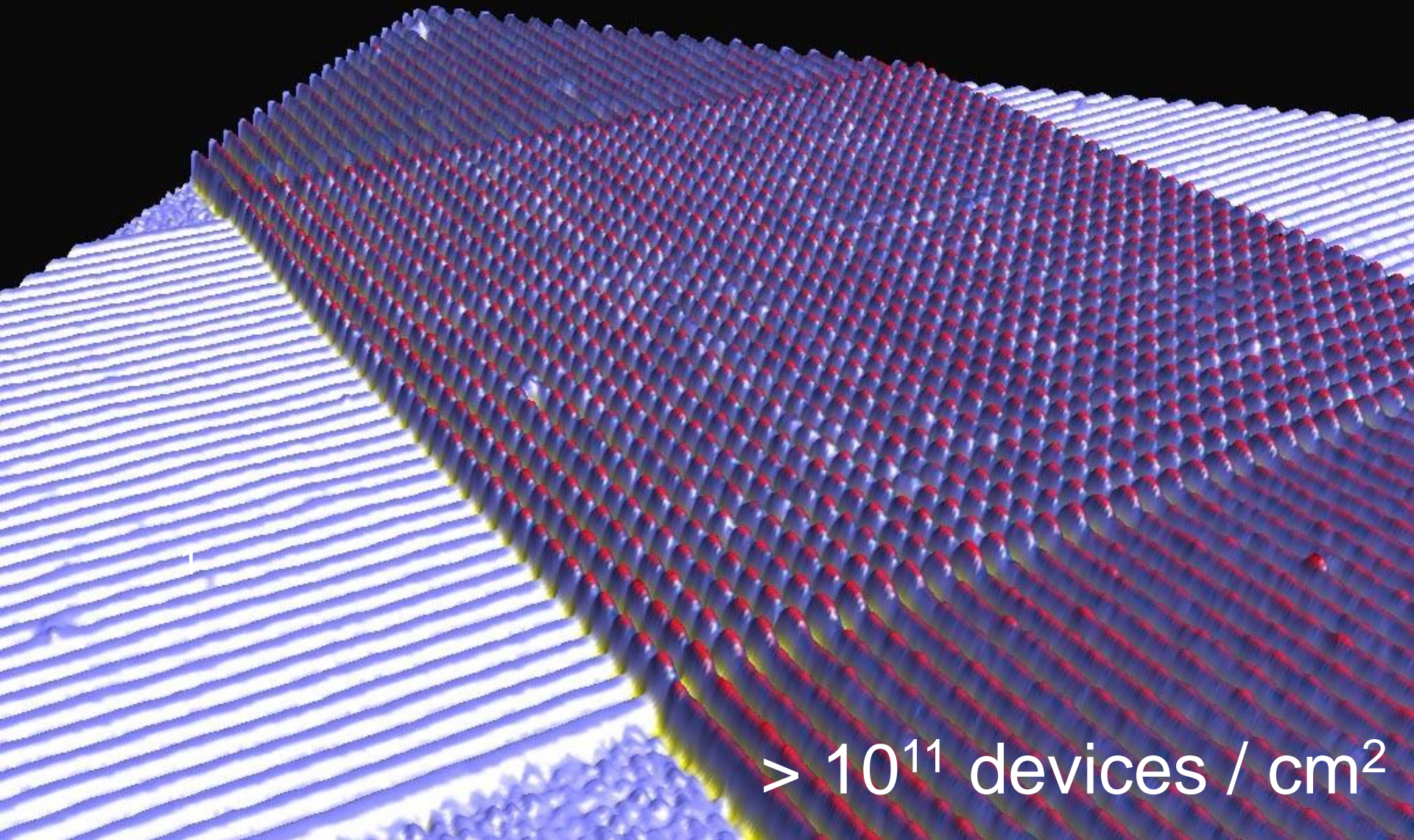
2. Adaptive resonance networks

(nano + CMOS)

3. Simulations

(fast, stable, incremental learning)

Orthogonal nanowire layers separated by memristive material



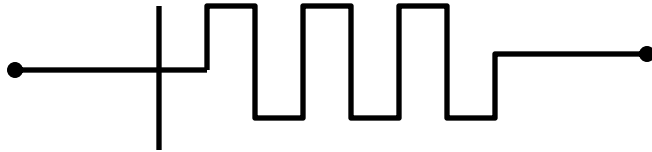
$> 10^{11}$ devices / cm^2

Definition: Memristive Device



schematic symbol

Definition: Memristive Device



$$\frac{d\mathbf{w}}{dt} = f(\mathbf{w}, v)$$

$$i = g(\mathbf{w}, v)v$$

or

$$\frac{d\mathbf{w}}{dt} = f(\mathbf{w}, i)$$

$$v = r(\mathbf{w}, i)i$$

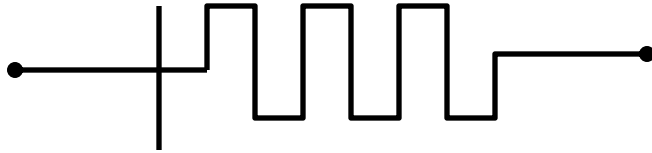


voltage-controlled



current-controlled

Definition: Memristive Device



state variables

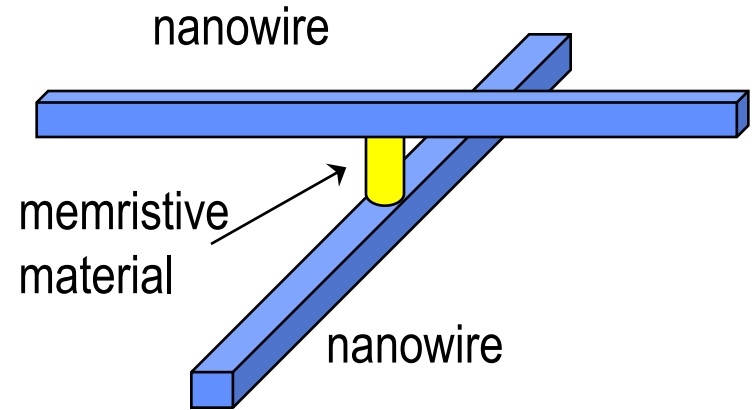
$$\frac{d\mathbf{w}}{dt} = f(\mathbf{w}, v)$$

$$i = g(\mathbf{w}, v)v$$

current

conductance

voltage

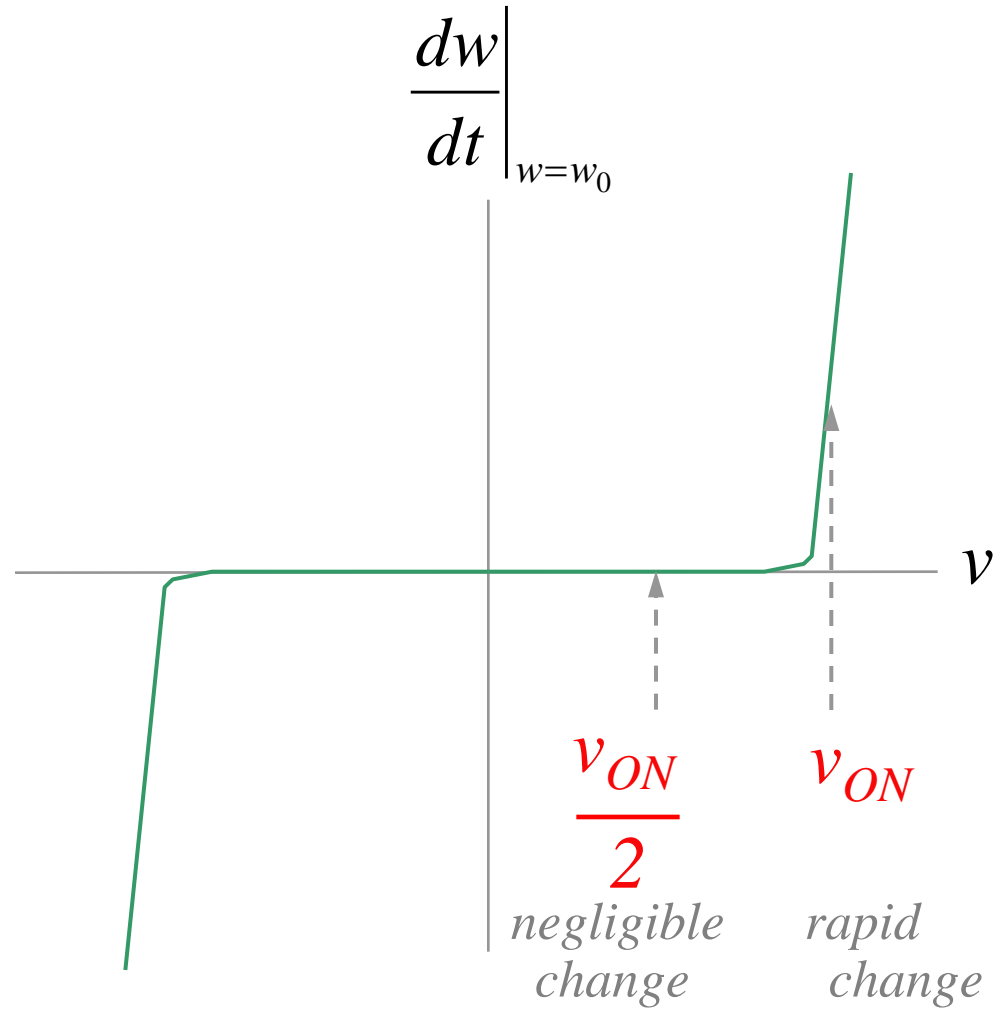


implementation

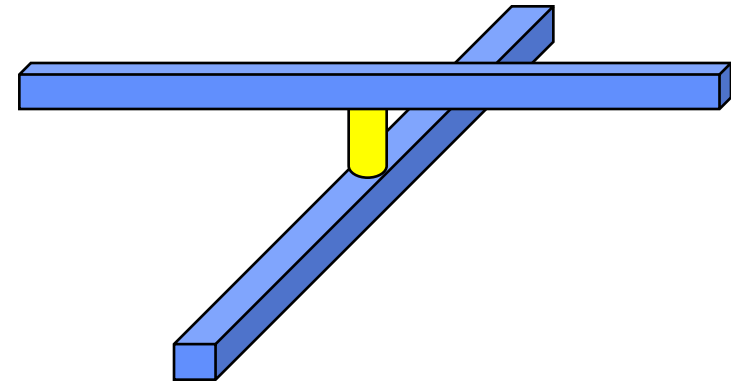
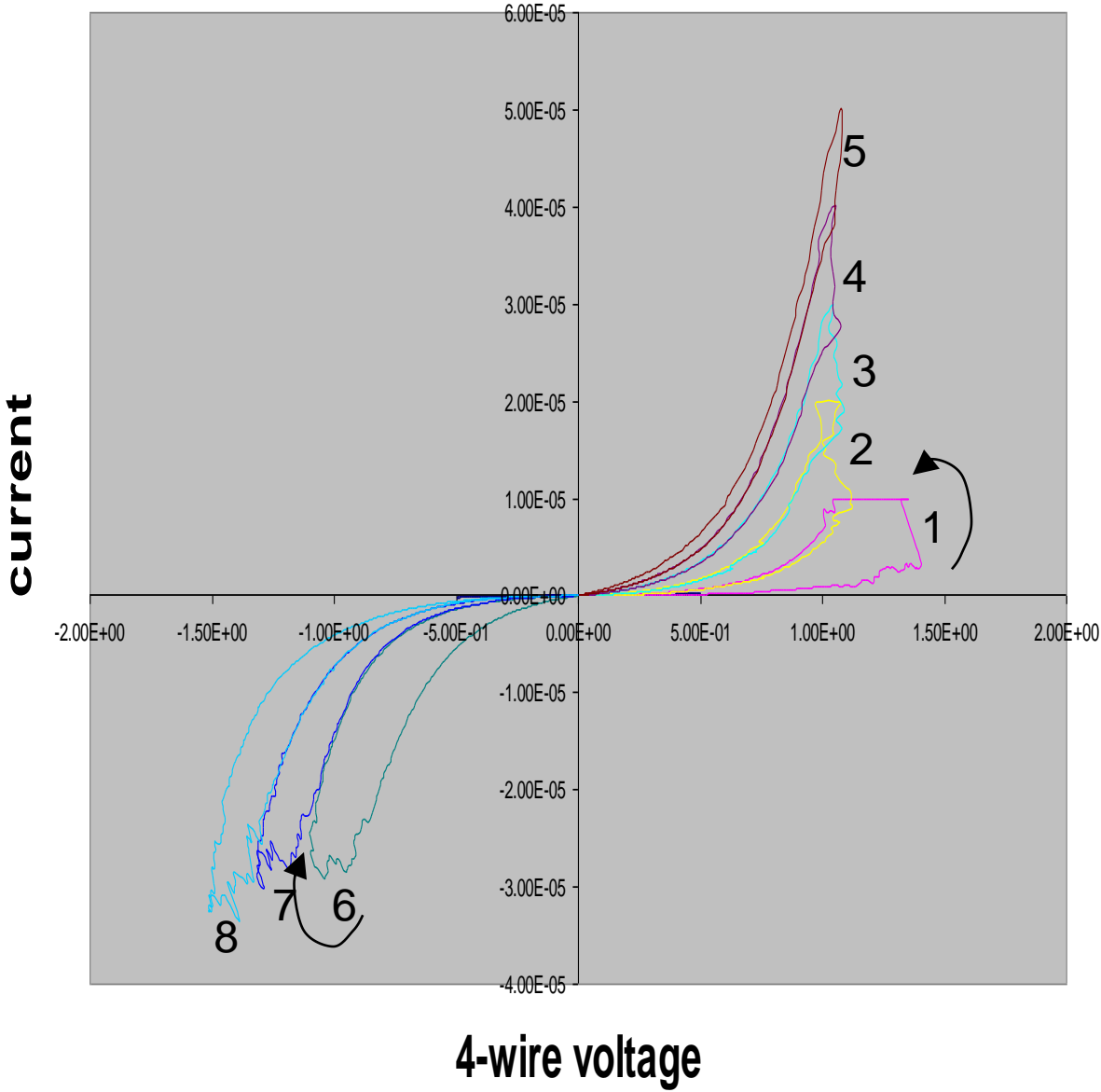
The exquisite nonlinearity of dw/dt

$$\frac{d\mathbf{w}}{dt} = f(\mathbf{w}, v)$$

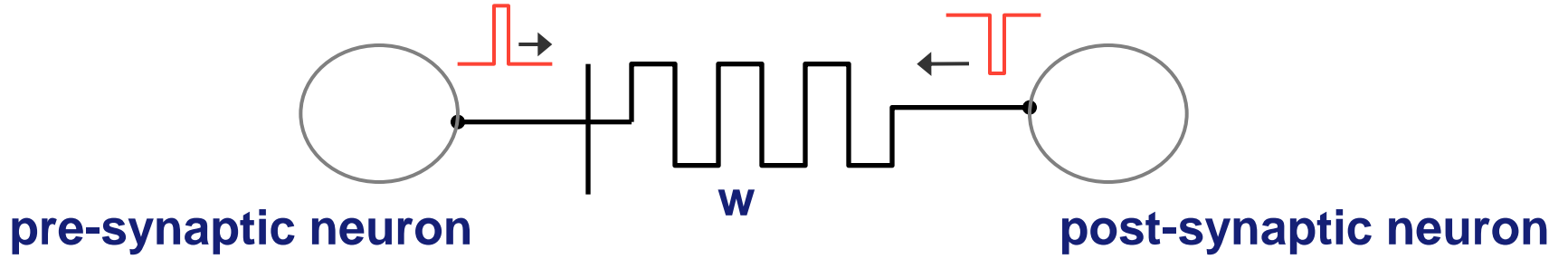
$$i = g(\mathbf{w}, v)v$$



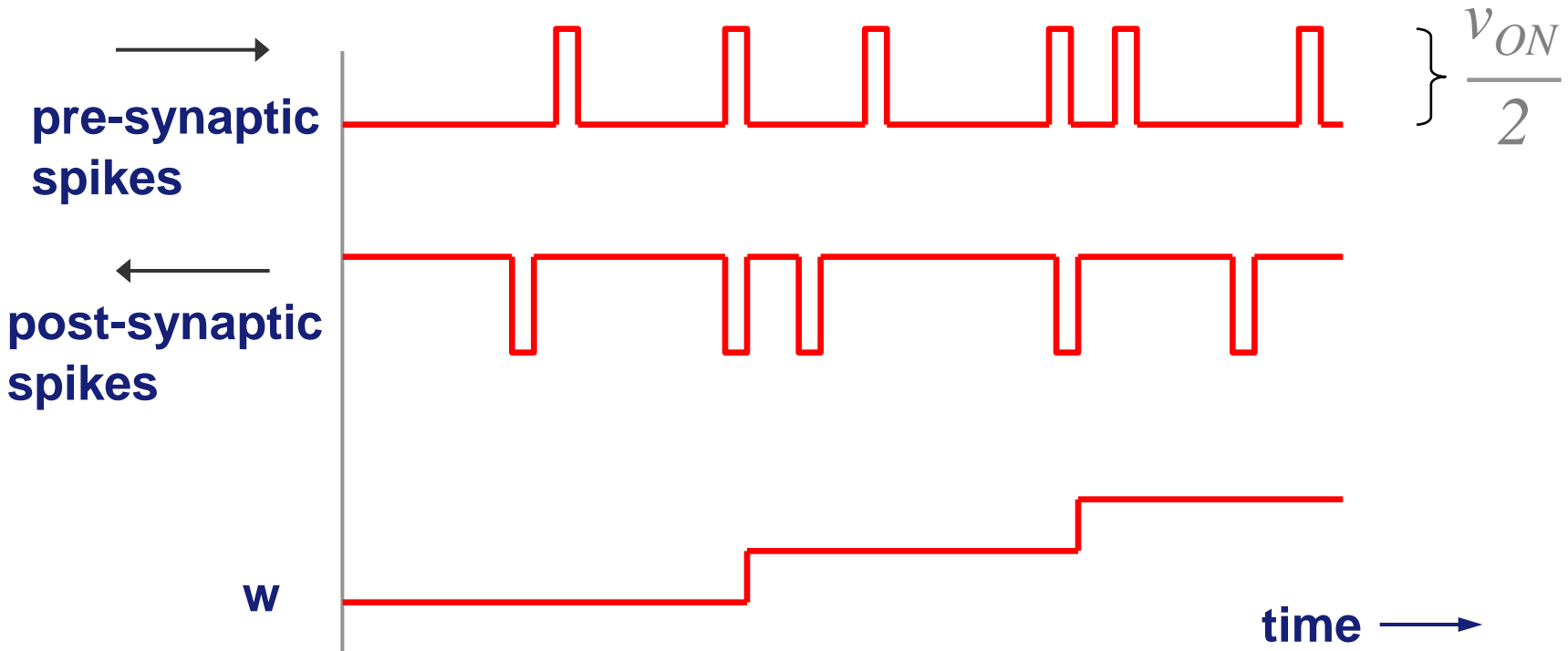
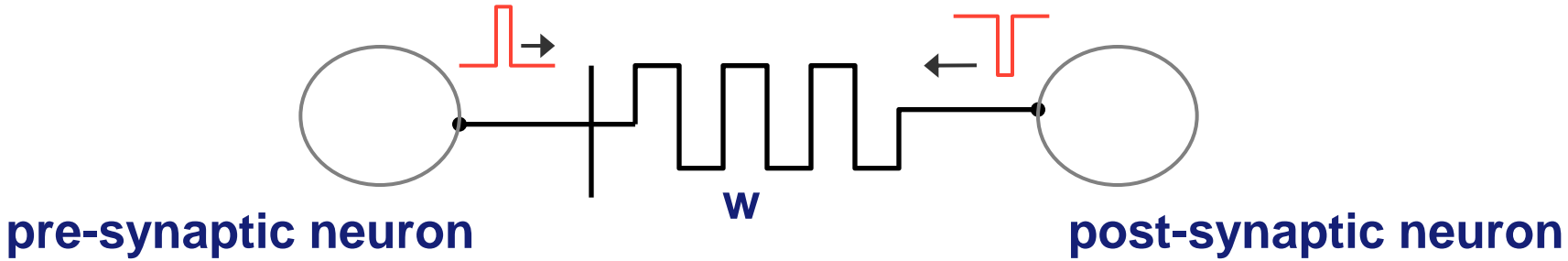
Experimental data



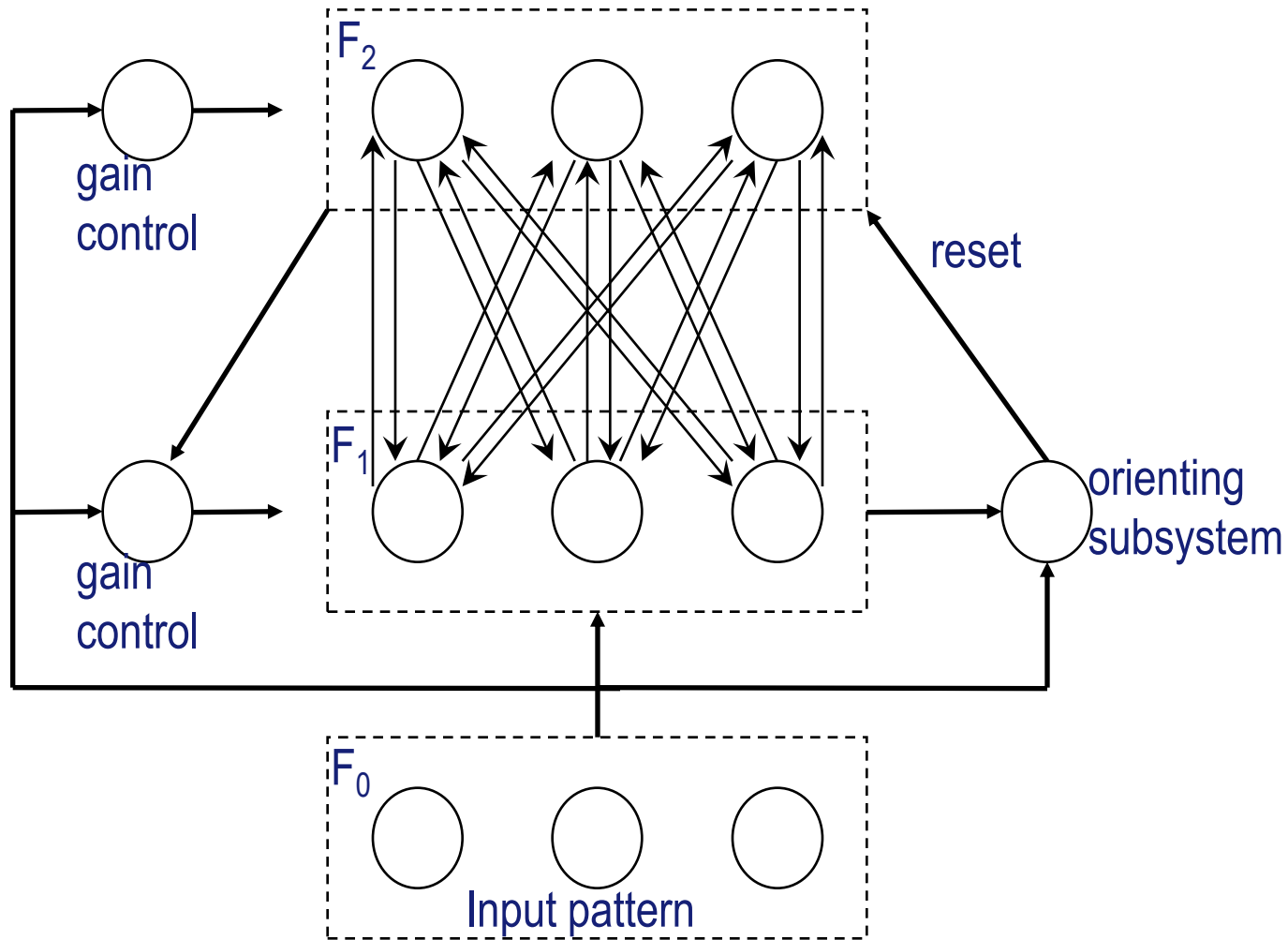
Constructive interference (Hebbian learning)



Constructive interference (Hebbian learning)



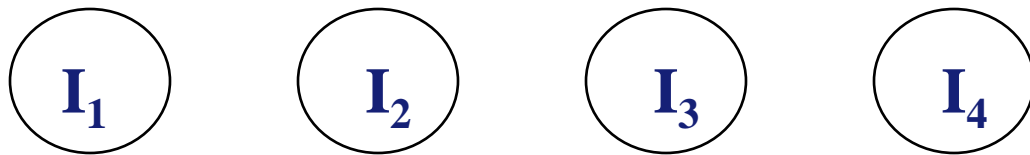
ART networks



Carpenter and Grossberg, "A Massively Parallel Architecture for a Self-organizing Neural Pattern Recognition Machine," *Computer Vision, Graphics, and Image Processing*, 1987

Adaptive Resonance Oversimplified¹

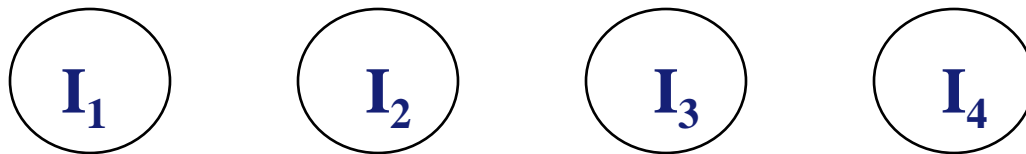
1. Initially no neurons in upper layer



watchdog
“O.S.”

Adaptive Resonance Oversimplified

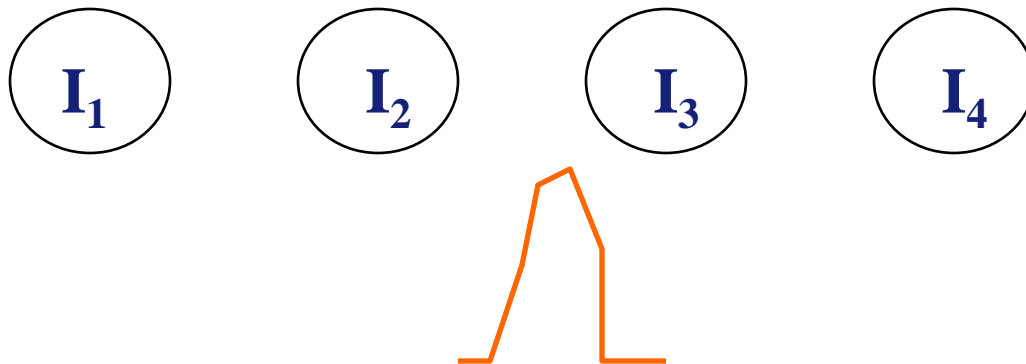
1. Initially no neurons in upper layer.
2. Input pattern comes in.



watchdog
“O.S.”

Adaptive Resonance Oversimplified

1. Initially no neurons in upper layer.
2. Input pattern comes in.



watchdog
“O.S.”

Adaptive Resonance Oversimplified

I_1

I_2

I_3

I_4



**watchdog
“O.S.”**

Adaptive Resonance Oversimplified

**Allocate
neuron!**



**watchdog
“O.S.”**

I₁

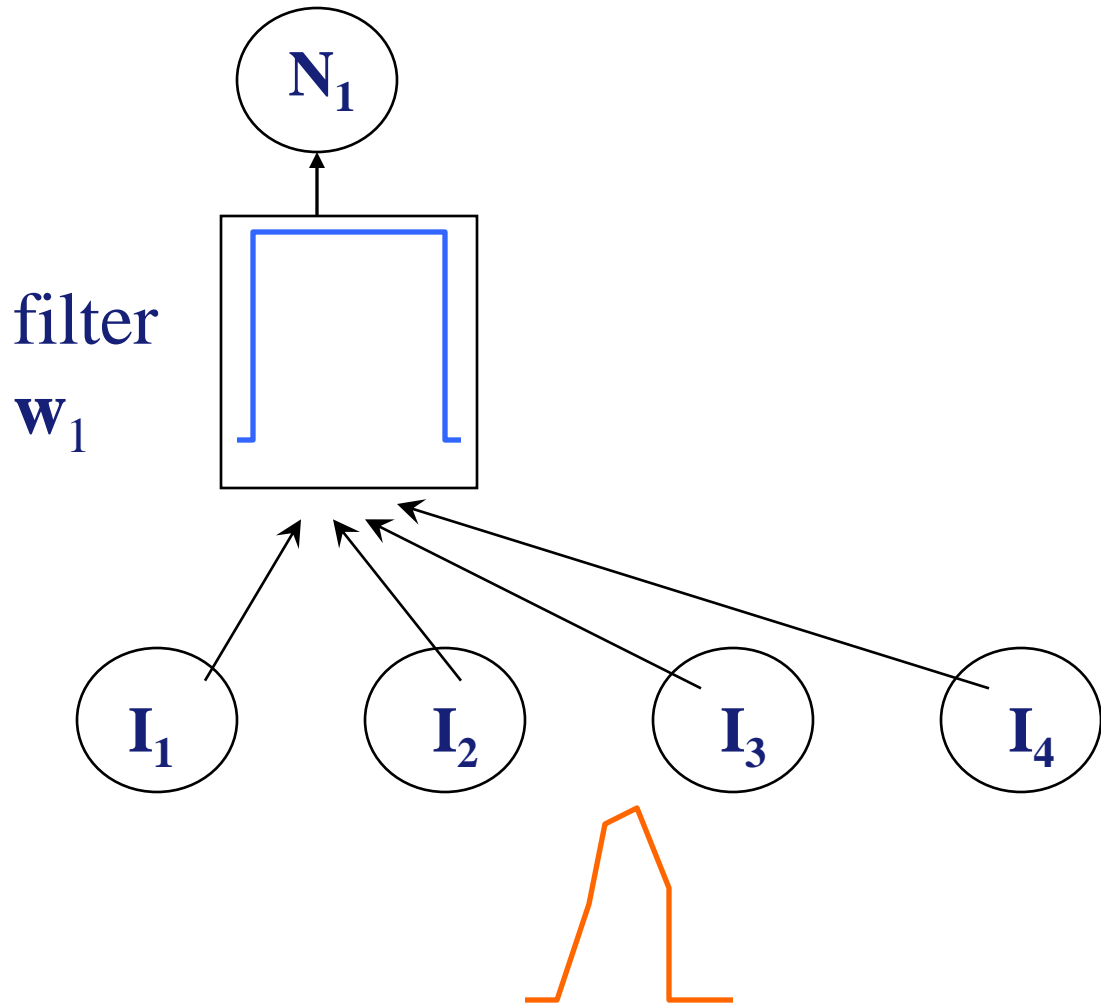
I₂

I₃

I₄

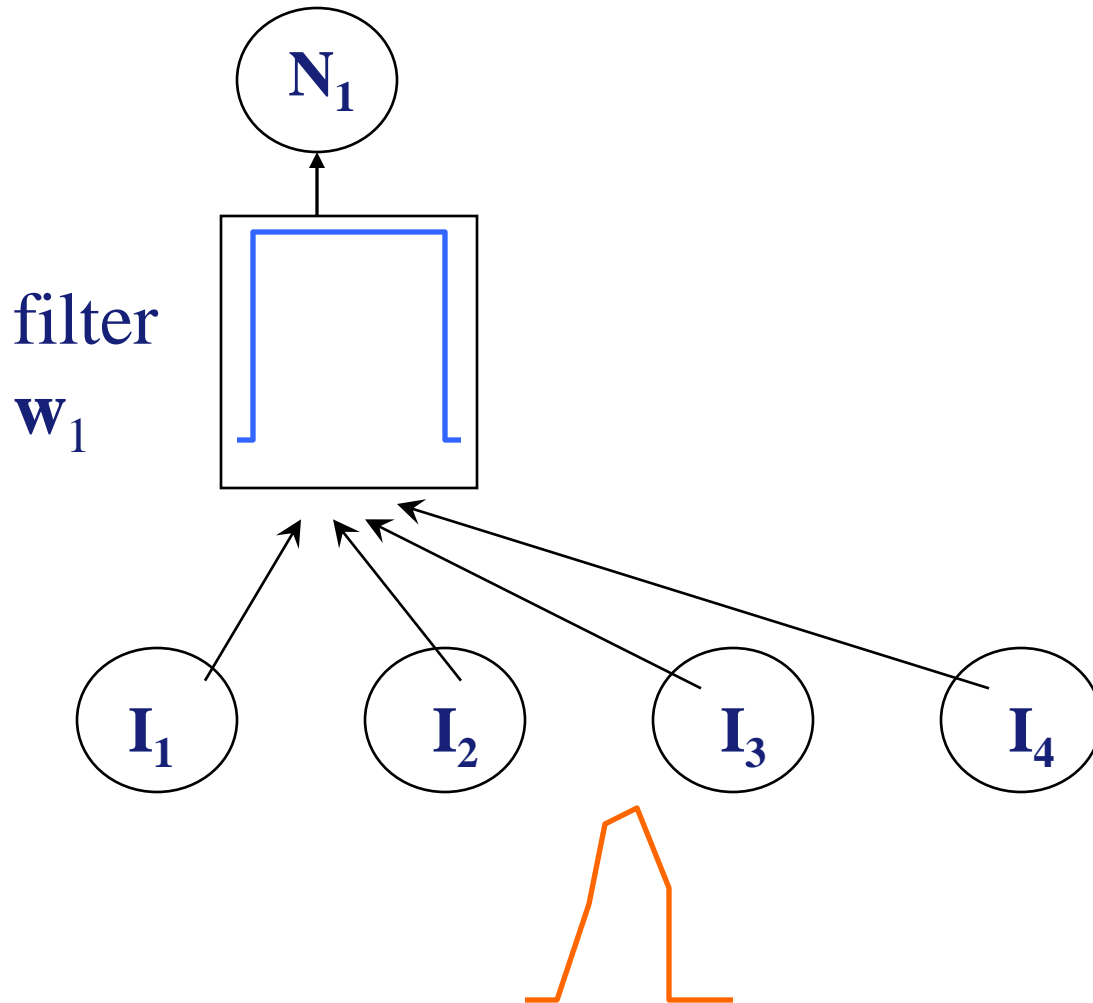


Adaptive Resonance Oversimplified



watchdog
"O.S."

Adaptive Resonance Oversimplified

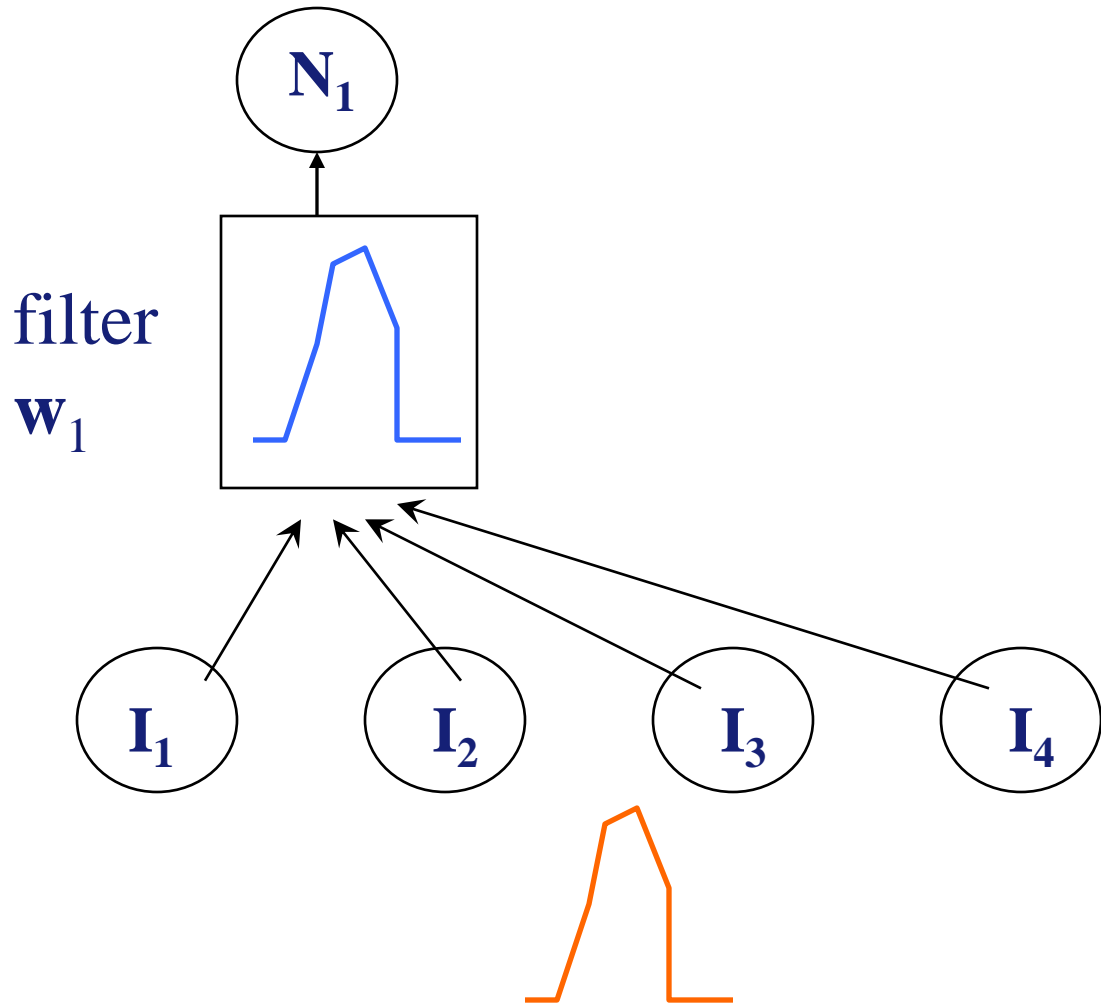


Learn pattern.



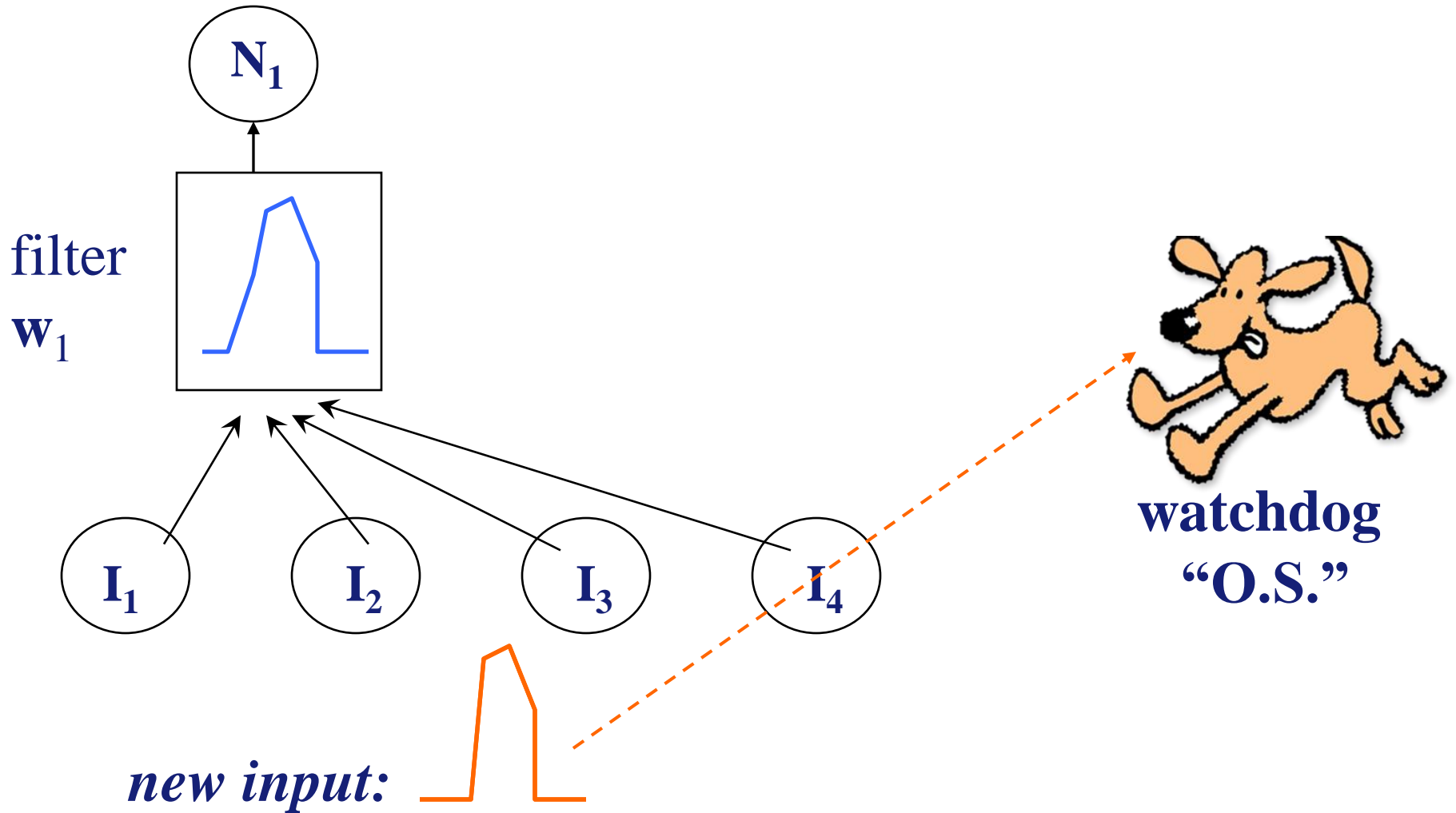
watchdog
"O.S."

Adaptive Resonance Oversimplified

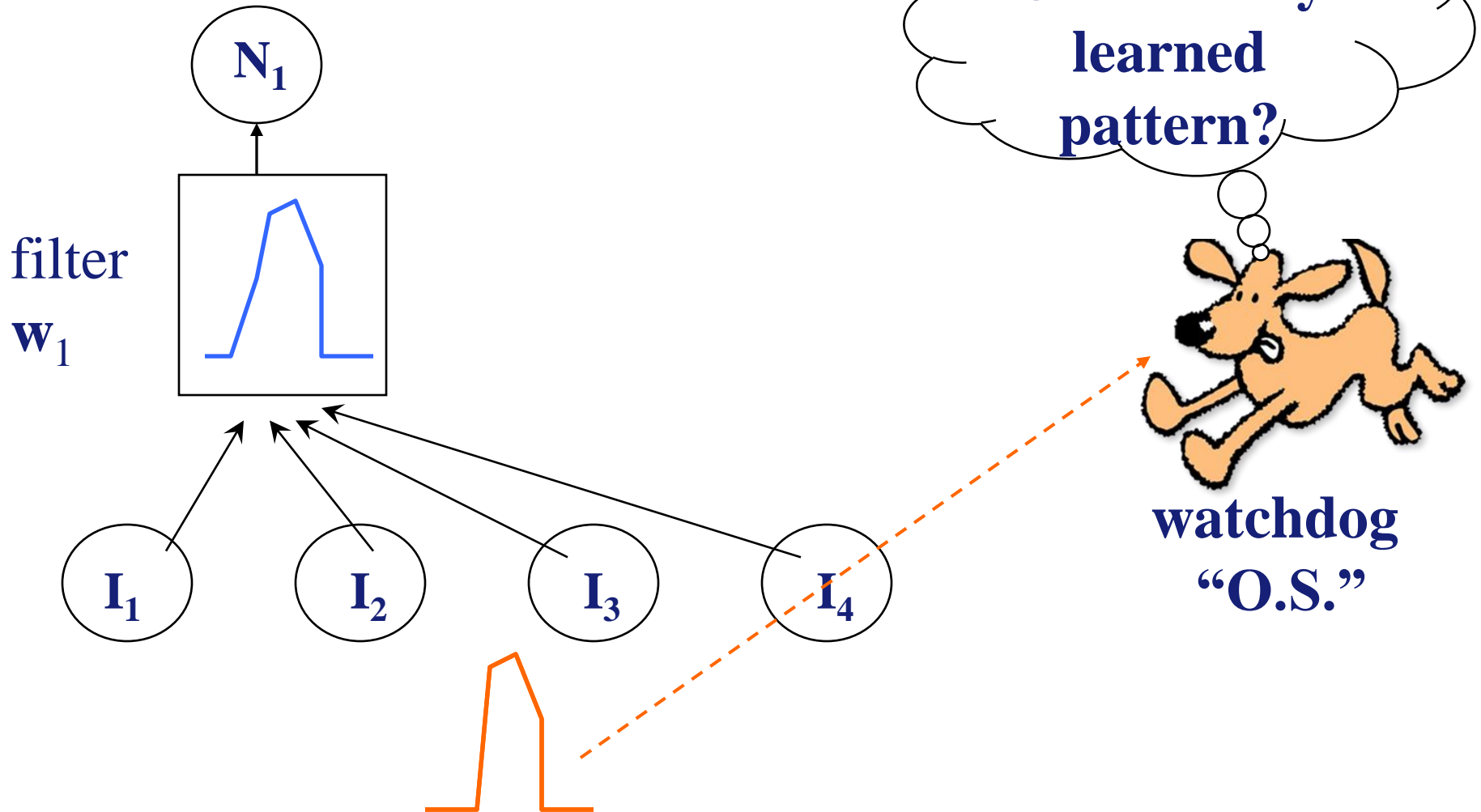


watchdog
“O.S.”

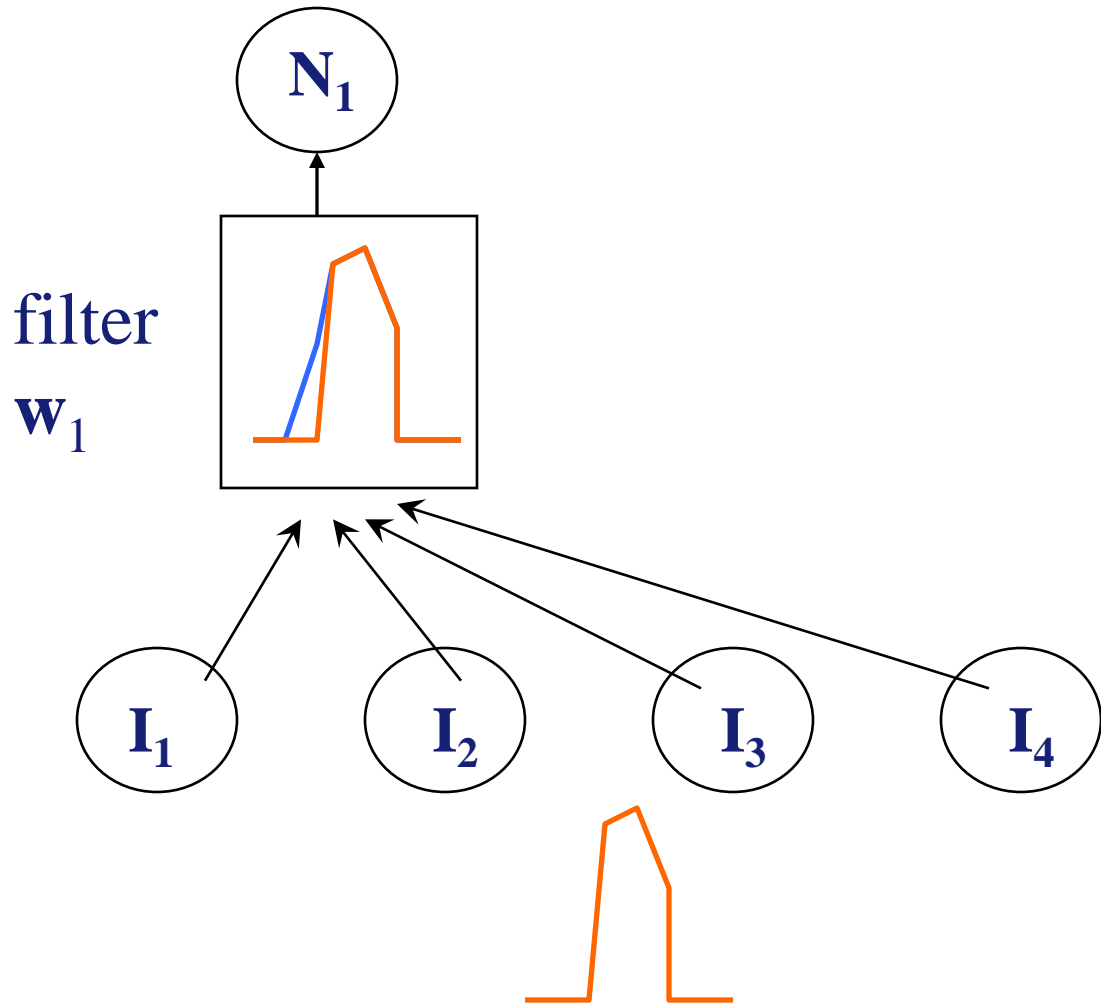
Adaptive Resonance Oversimplified



Adaptive Resonance Oversimplified

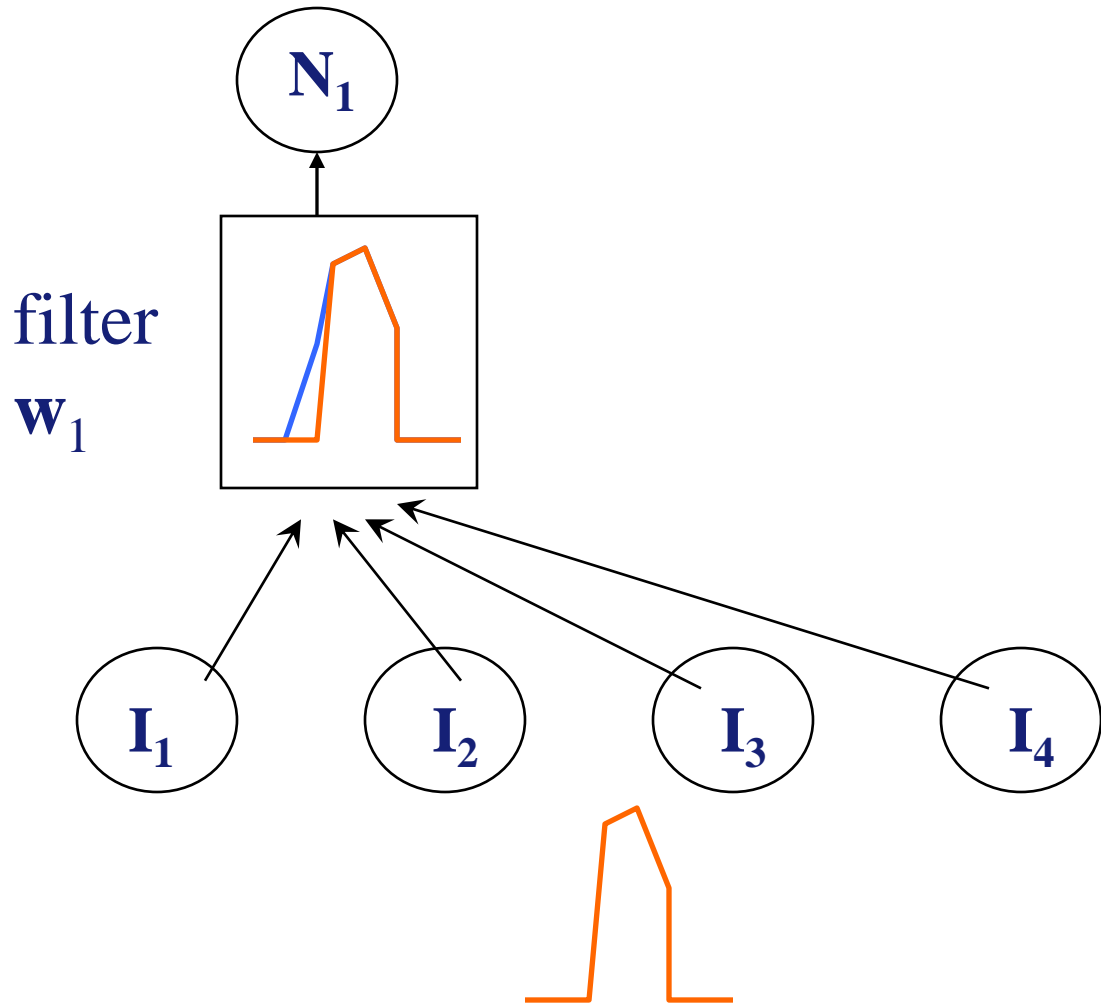


Adaptive Resonance Oversimplified



watchdog
"O.S."

Adaptive Resonance Oversimplified

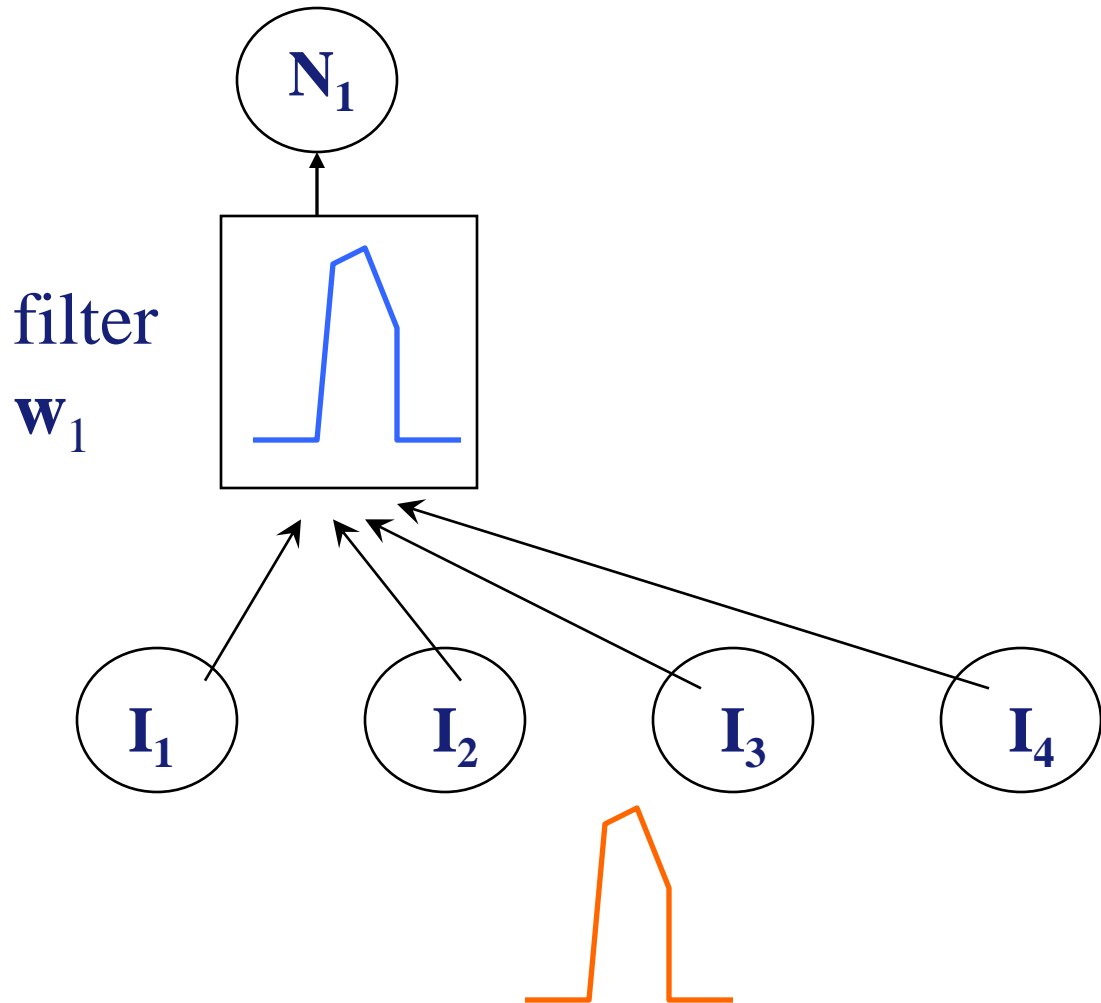


**Close enough.
Refine!**



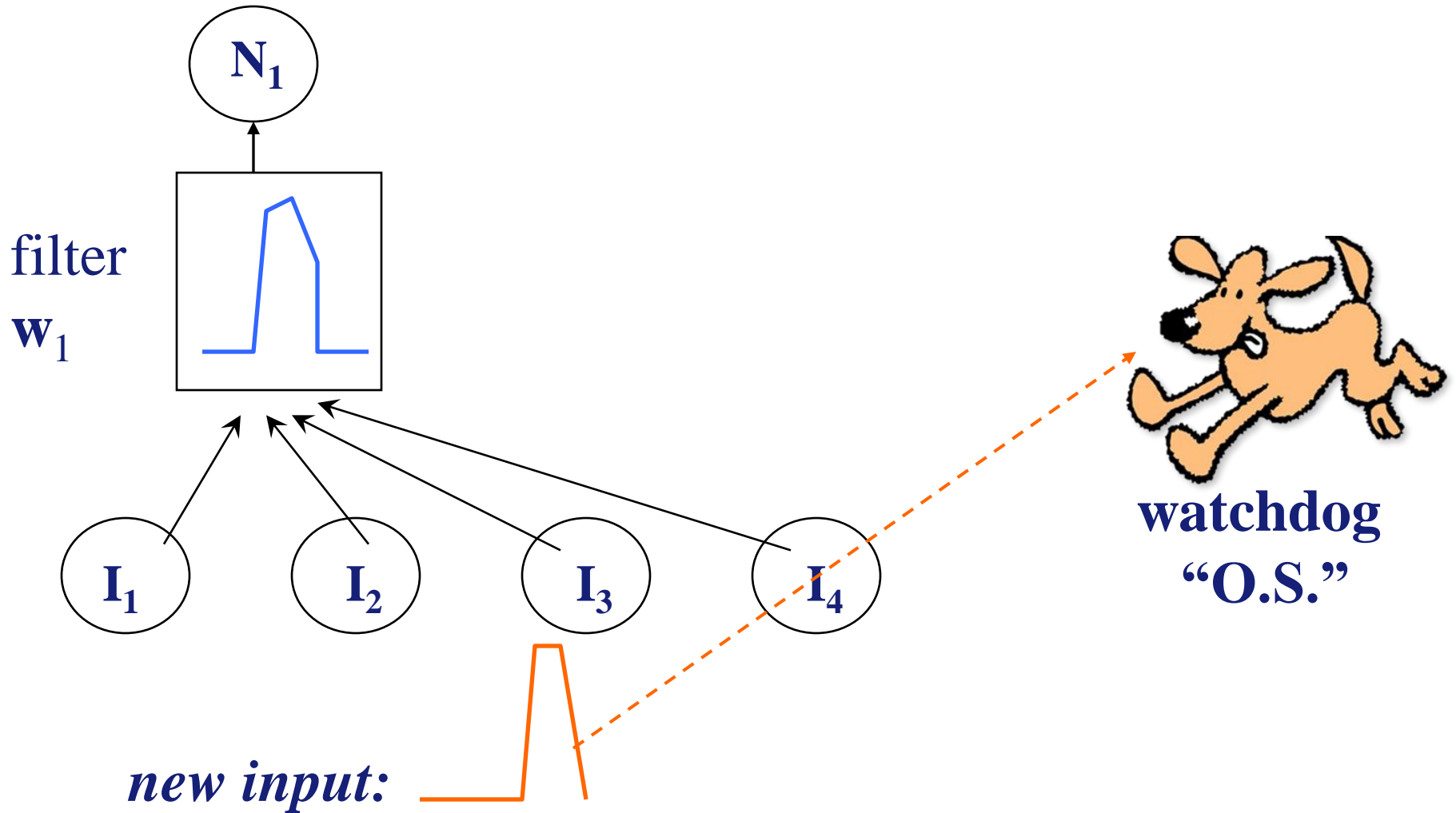
**watchdog
"O.S."**

Adaptive Resonance Oversimplified

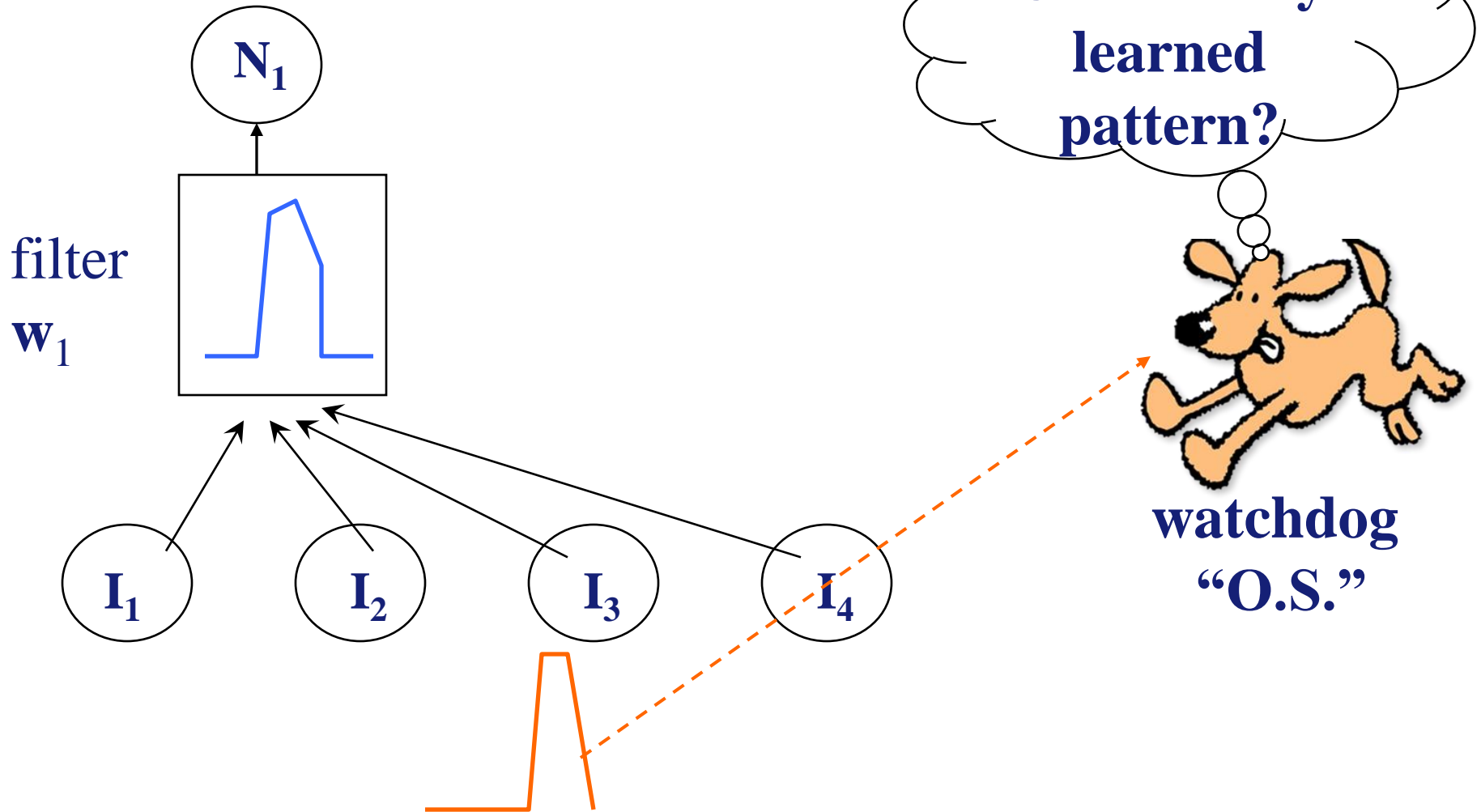


watchdog
"O.S."

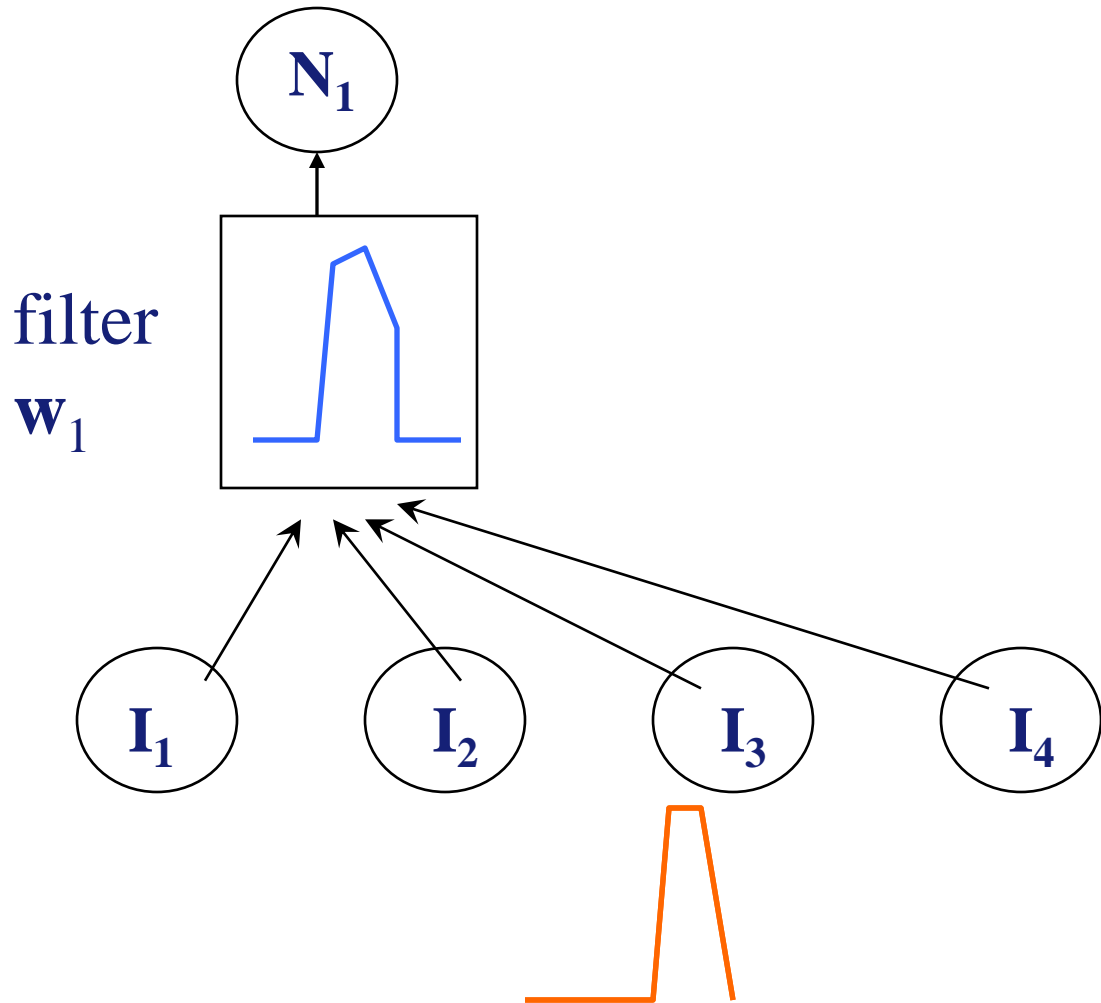
Adaptive Resonance Oversimplified



Adaptive Resonance Oversimplified



Adaptive Resonance Oversimplified

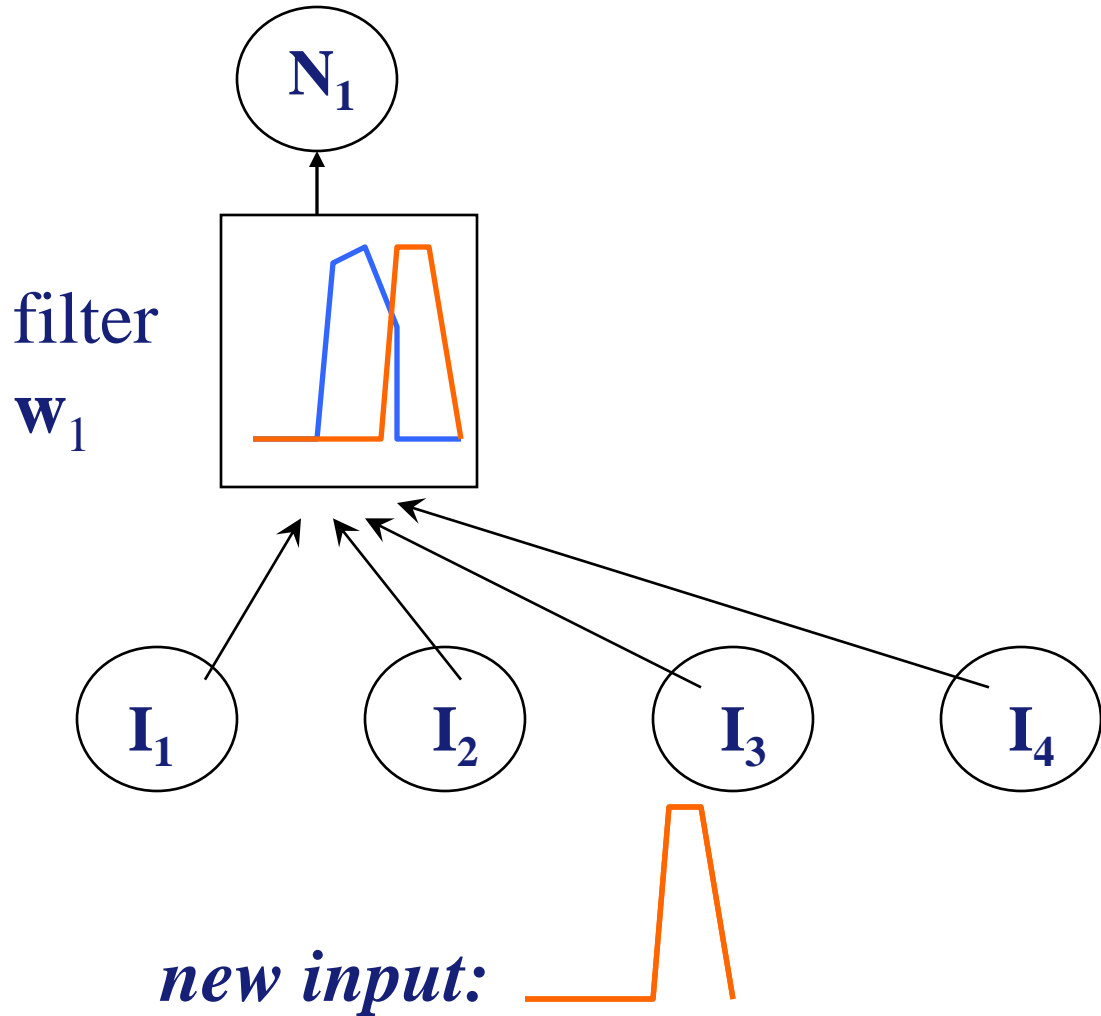


Close to any
learned
pattern?



watchdog
“O.S.”

Adaptive Resonance Oversimplified

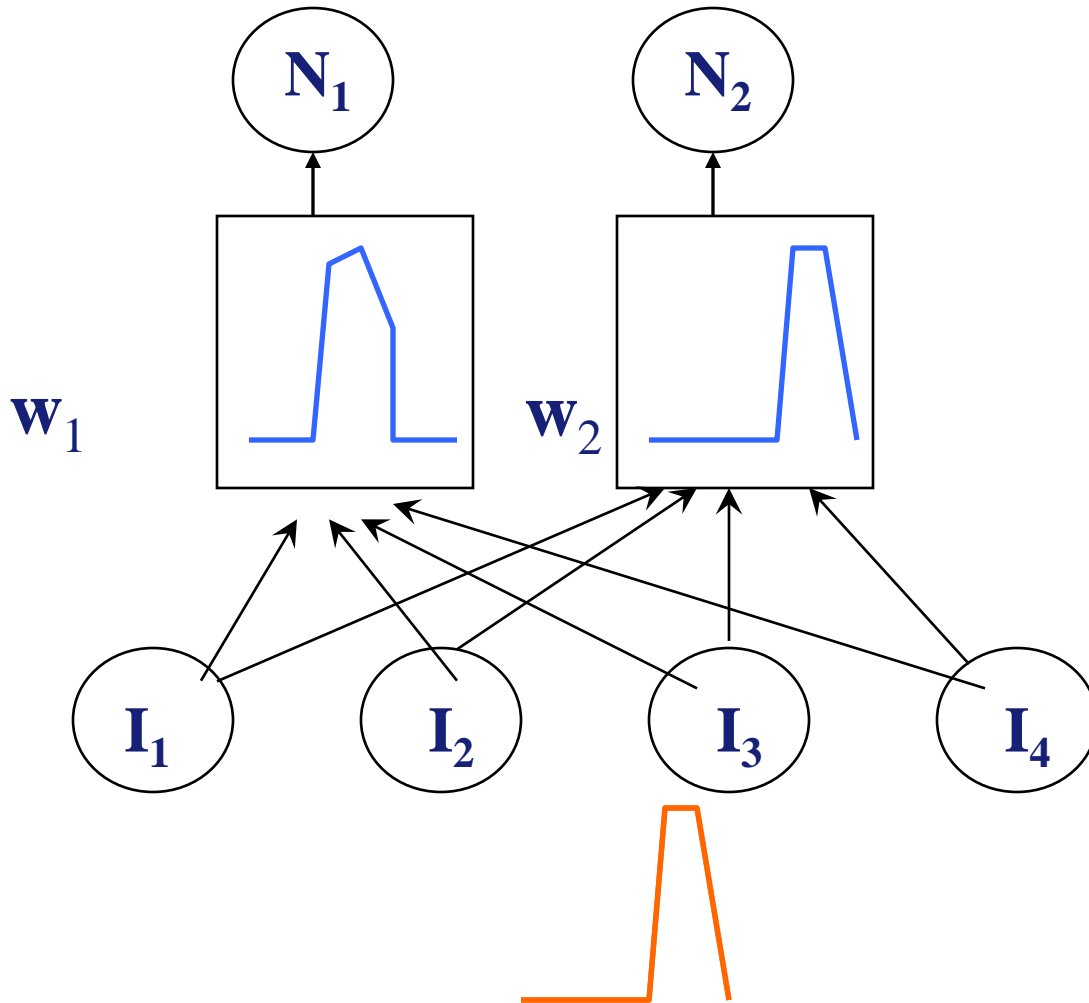


No. Allocate
and learn.



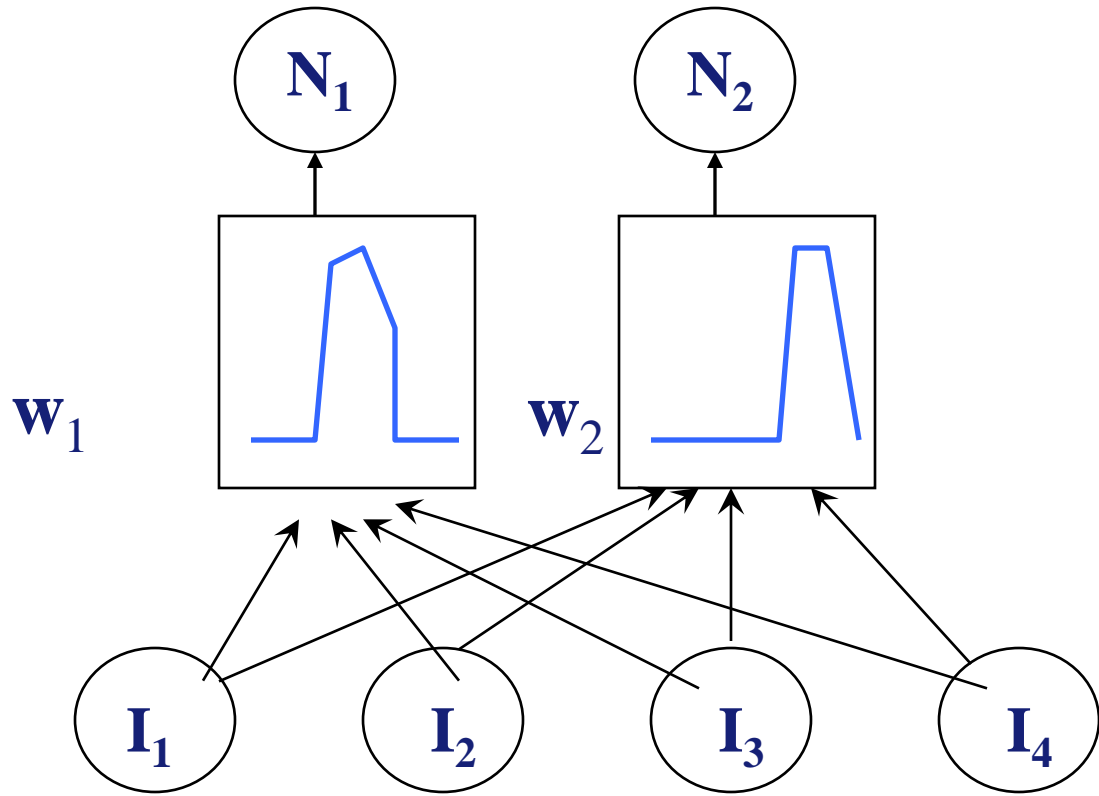
watchdog
"O.S."

Adaptive Resonance Oversimplified



watchdog
“O.S.”

Adaptive Resonance Oversimplified



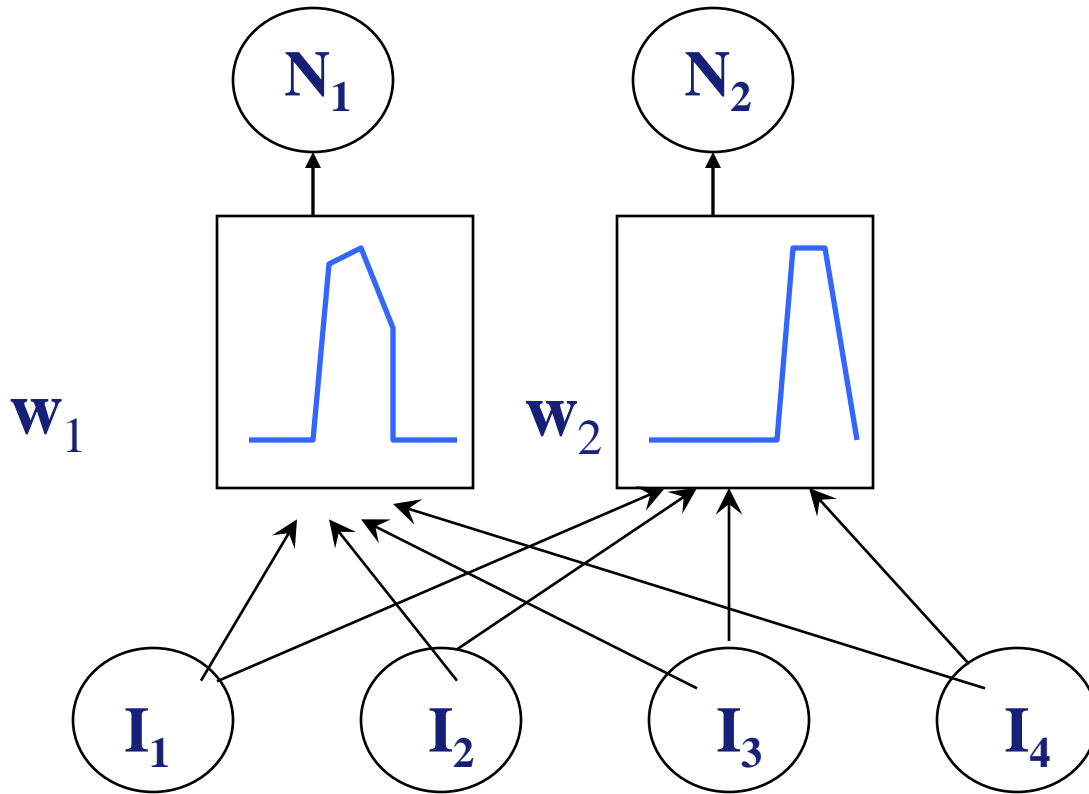
Winner =

$$\arg \max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

Adaptive Resonance Oversimplified



Winner =

$$\arg \max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

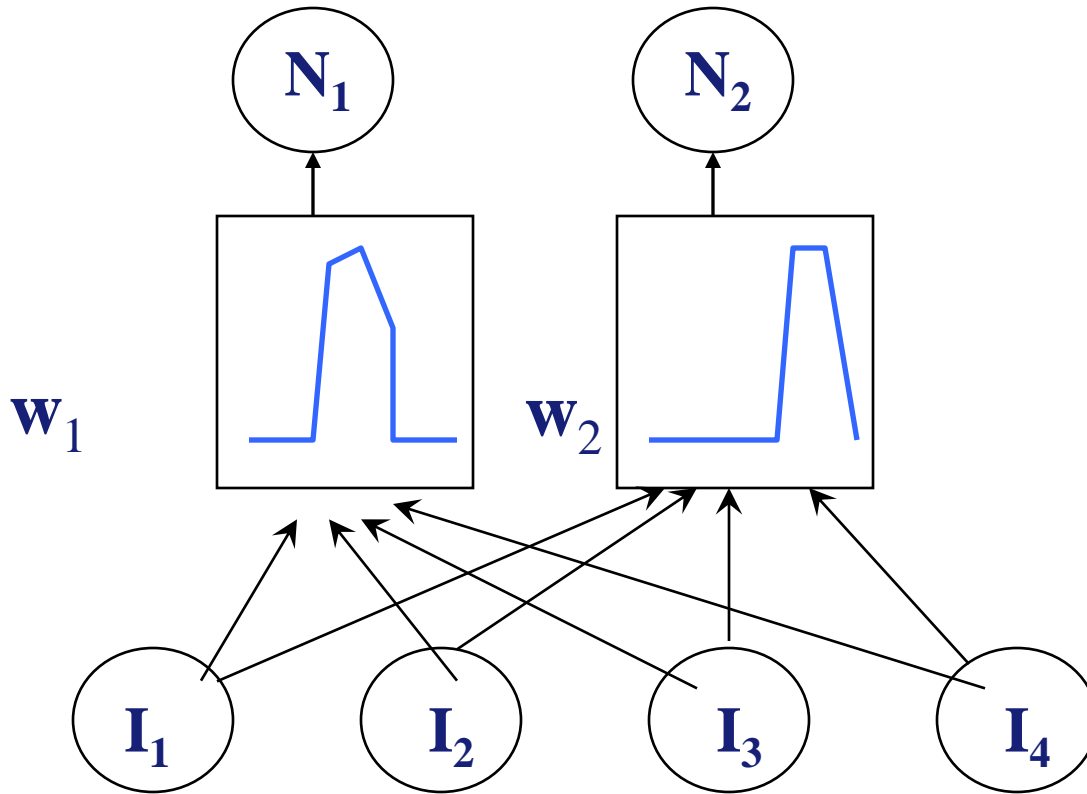
$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

Vigilance:

1.0 \rightarrow perfect fit

0.0 \rightarrow anything goes

Adaptive Resonance Oversimplified



If winner
resonate and refine learned pattern
otherwise
allocate new neuron

Winner =

$$\arg \max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

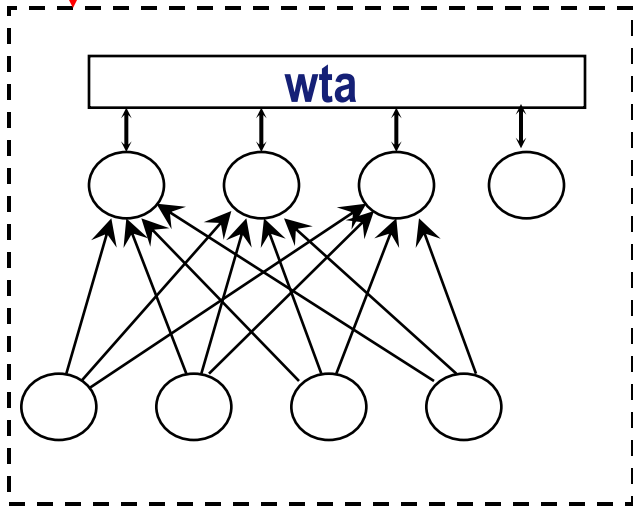
$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

Vigilance:
1.0 → perfect fit
0.0 → anything goes

Generalization with vigilance, ρ

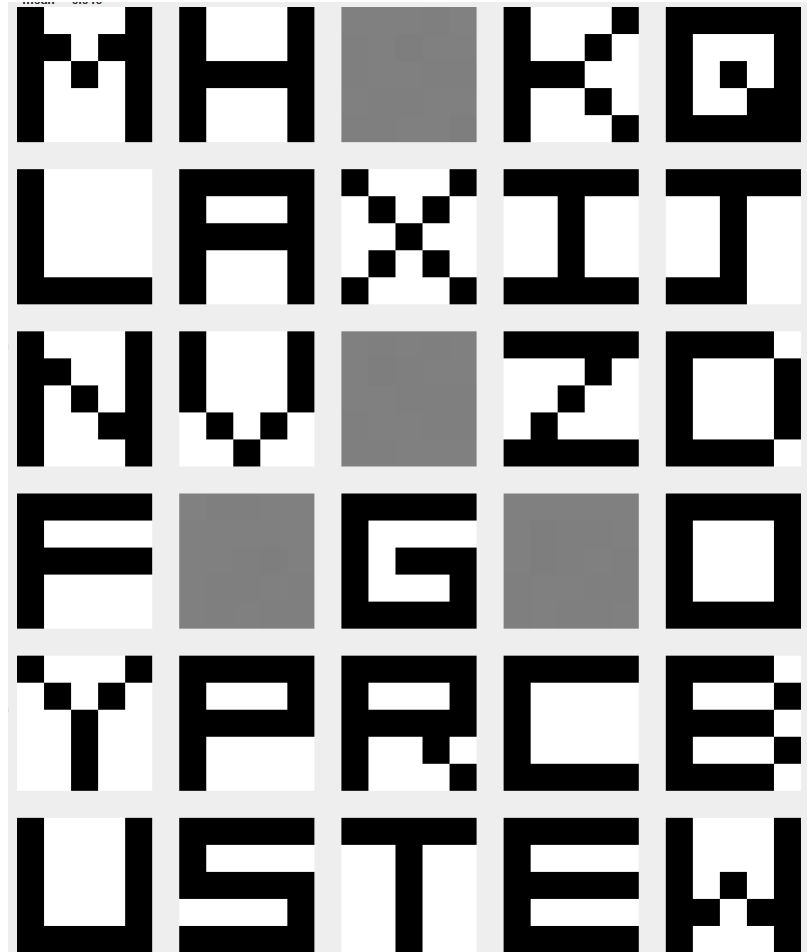
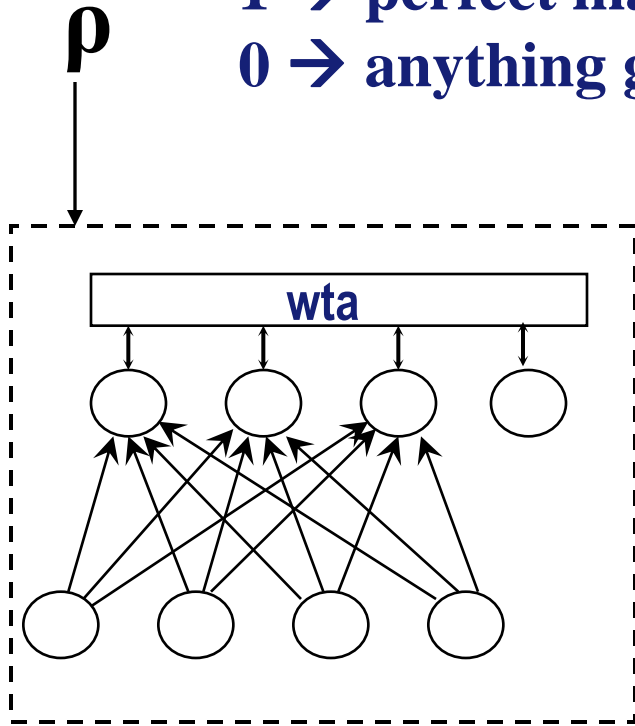
1 \rightarrow perfect match
0 \rightarrow anything goes

ρ



Generalization with vigilance, ρ

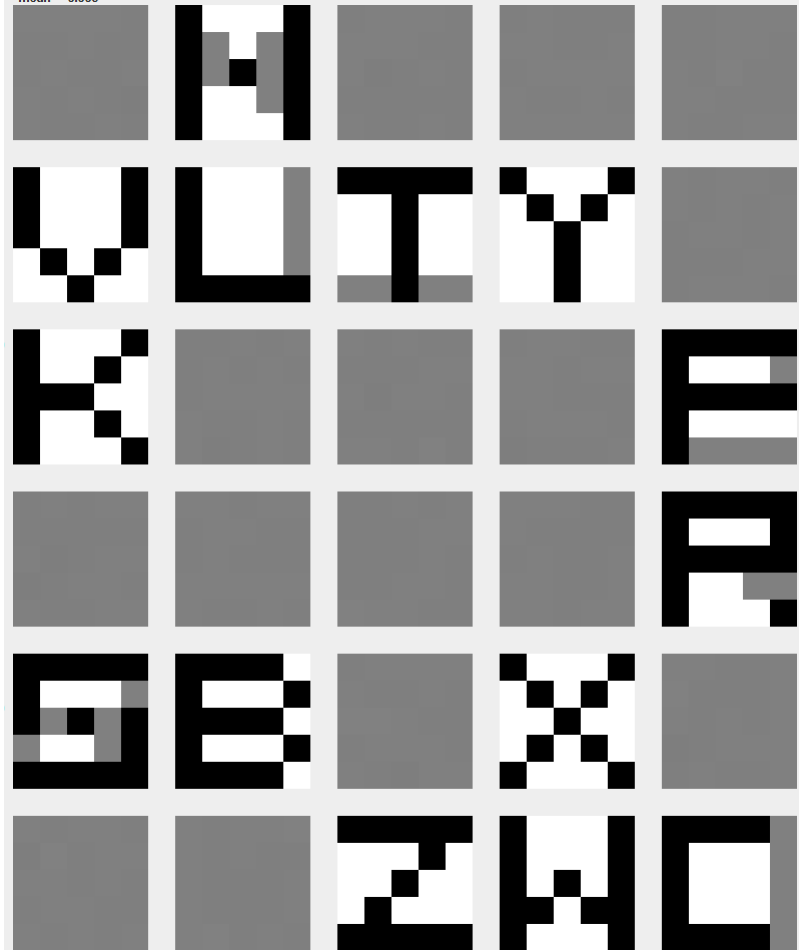
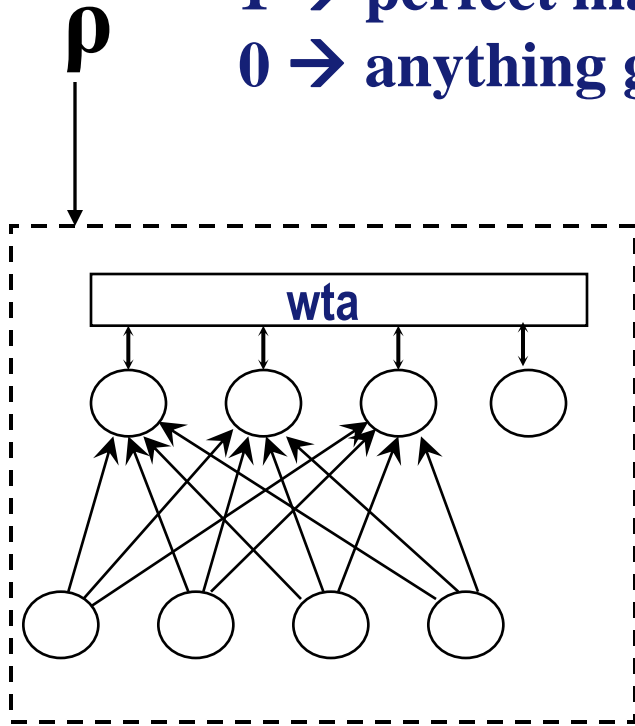
1 \rightarrow perfect match
0 \rightarrow anything goes



$\rho = 0.99$

Generalization with vigilance, ρ

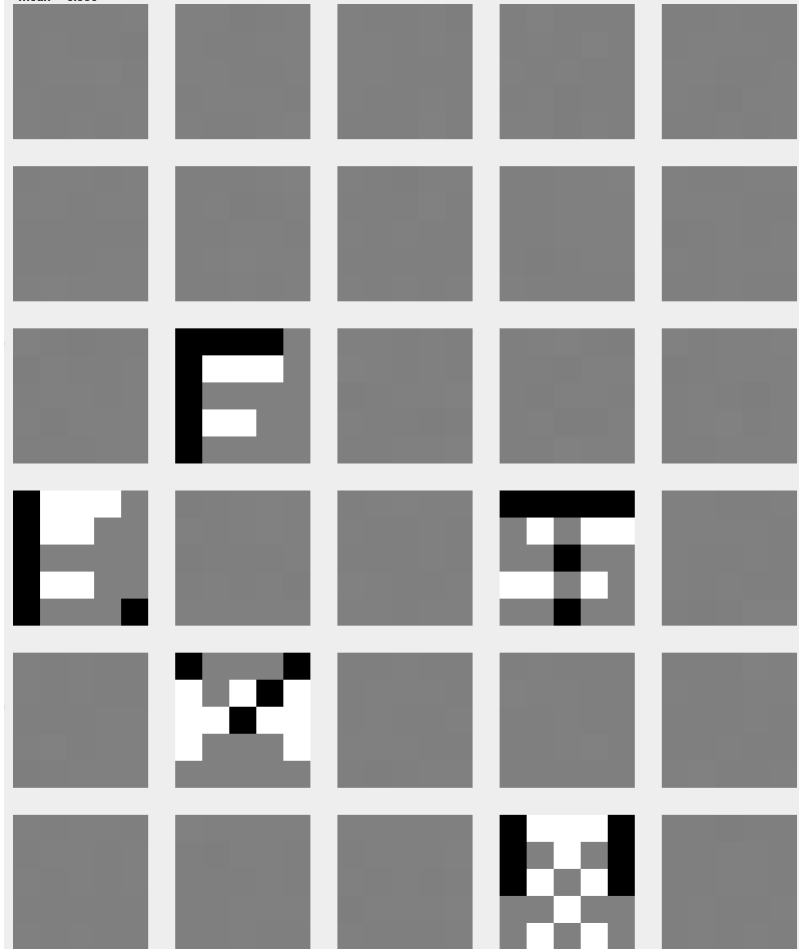
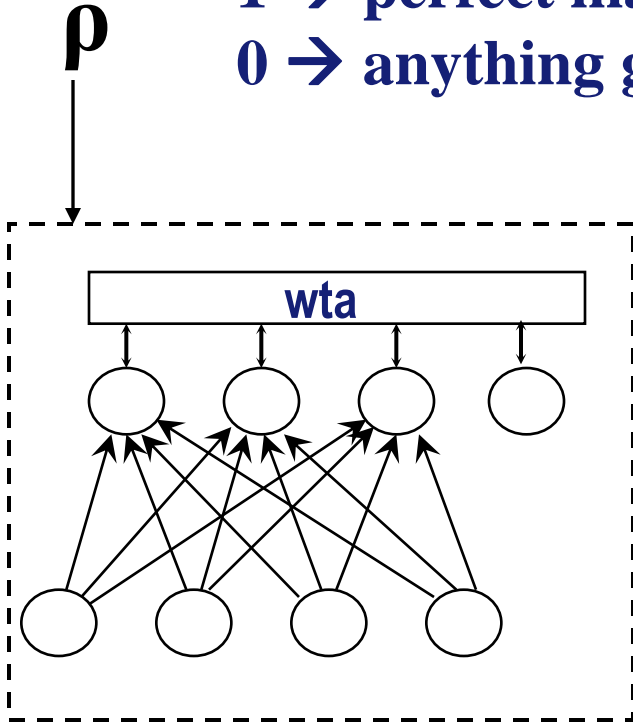
1 \rightarrow perfect match
0 \rightarrow anything goes



$\rho = 0.8$

Generalization with vigilance, ρ

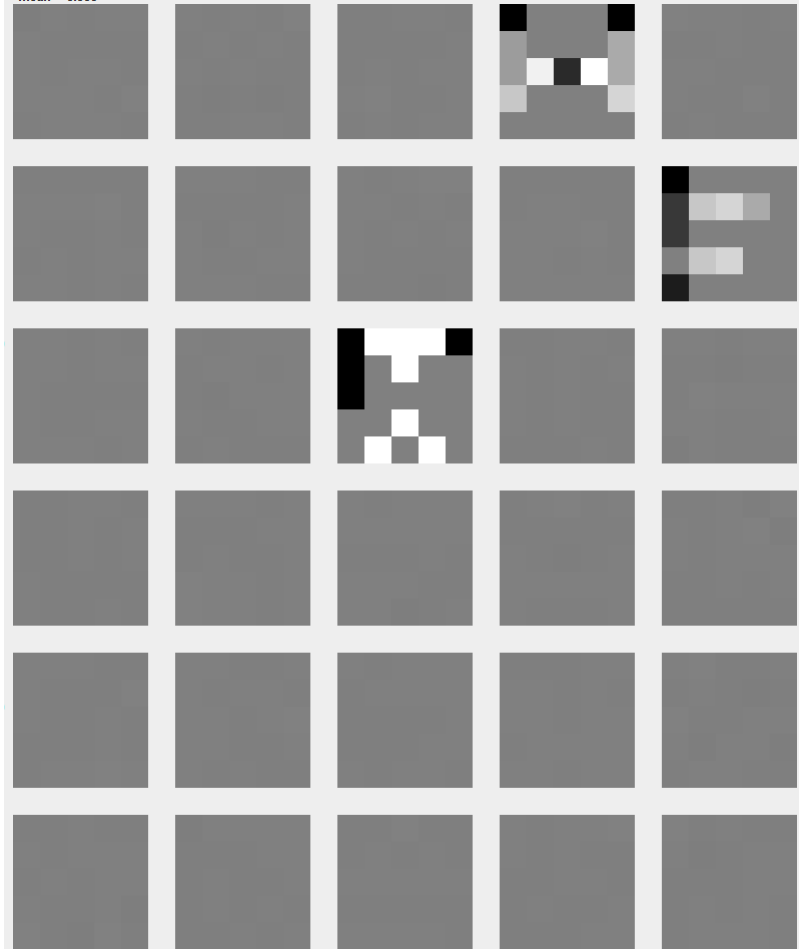
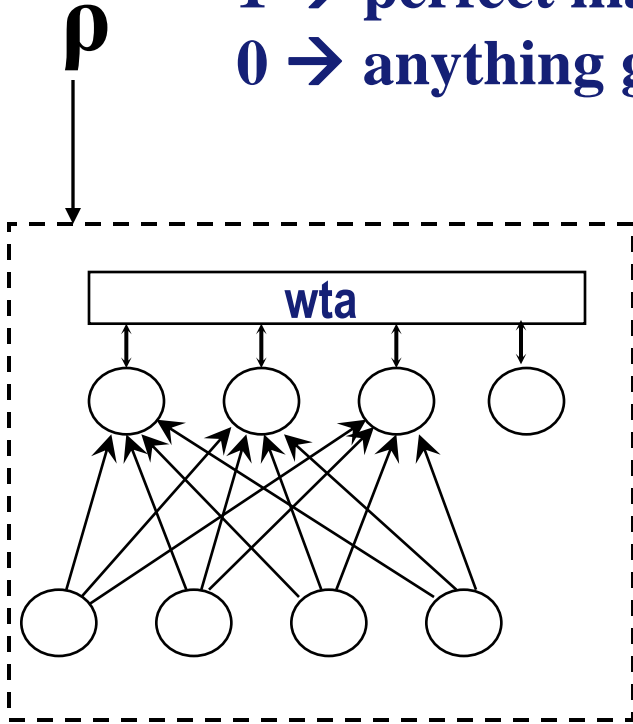
1 \rightarrow perfect match
0 \rightarrow anything goes



$\rho = 0.5$

Generalization with vigilance, ρ

1 \rightarrow perfect match
0 \rightarrow anything goes

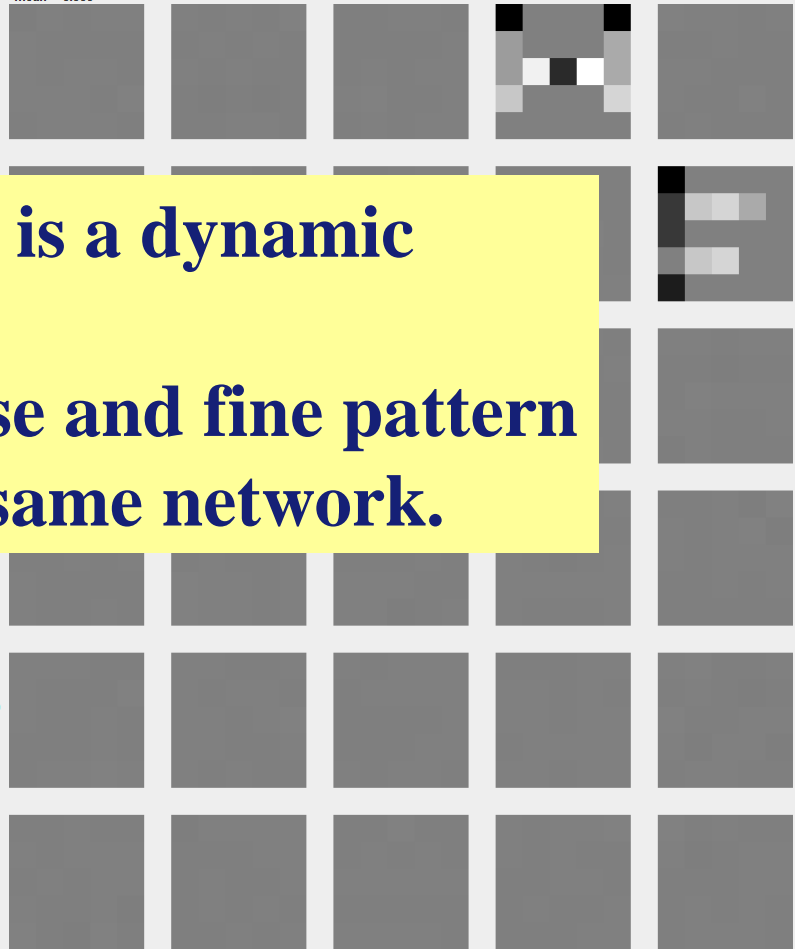
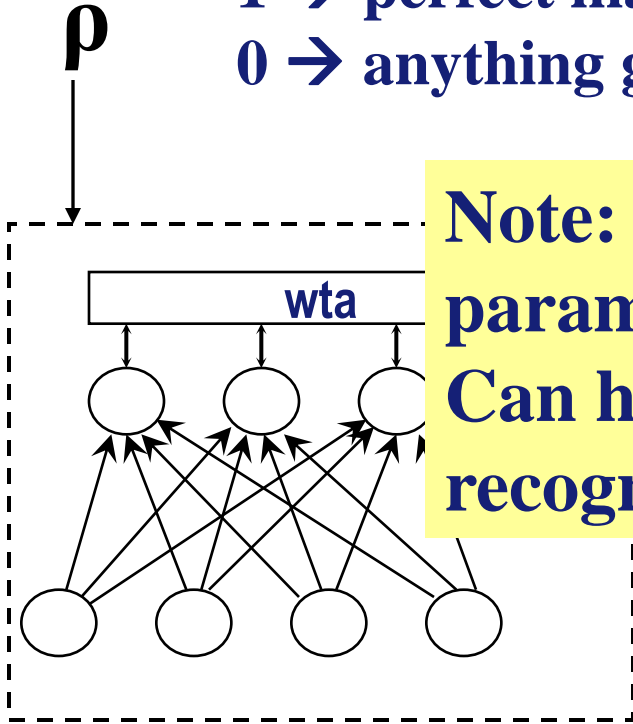


$\rho = 0.2$

Generalization with vigilance, ρ

1 \rightarrow perfect match
0 \rightarrow anything goes

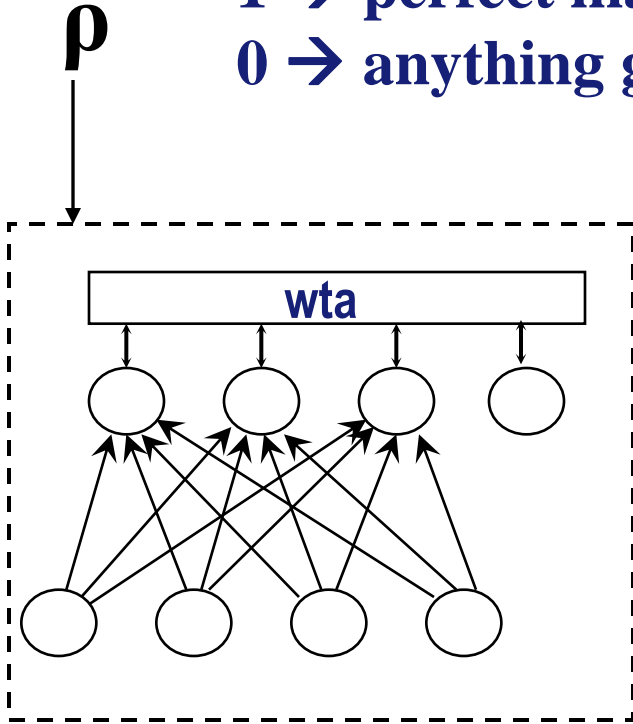
**Note: vigilance is a dynamic parameter.
Can have coarse and fine pattern recognition in same network.**



$\rho = 0.2$

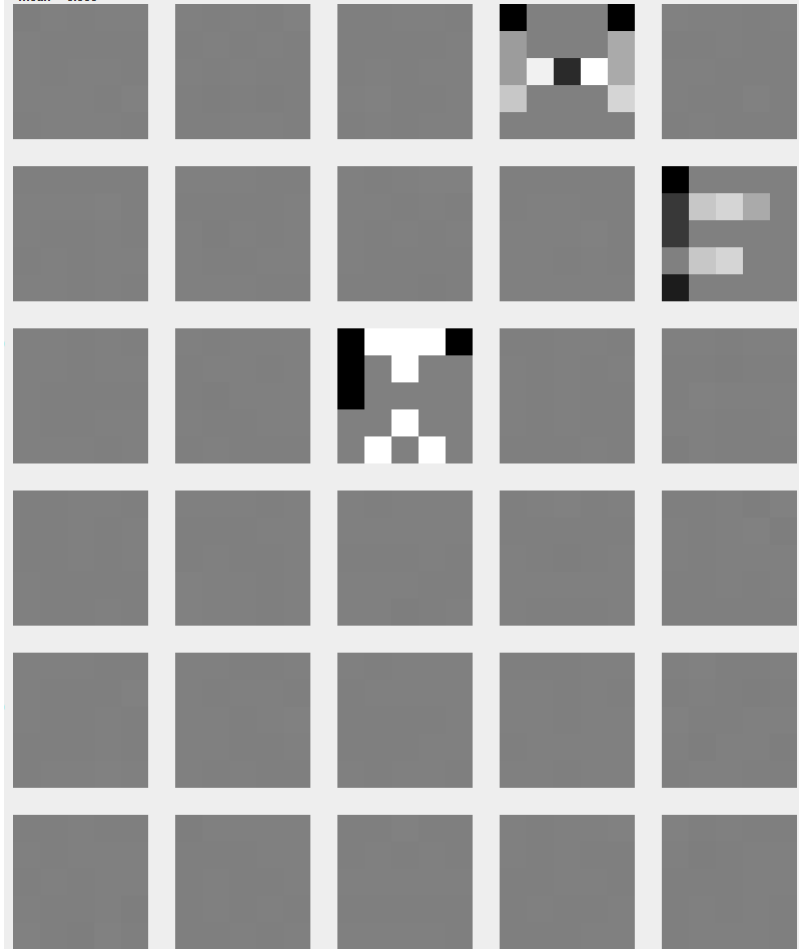
Generalization with vigilance, ρ

1 \rightarrow perfect match
0 \rightarrow anything goes



$\rho = 0.2$: A B C ... X Y Z

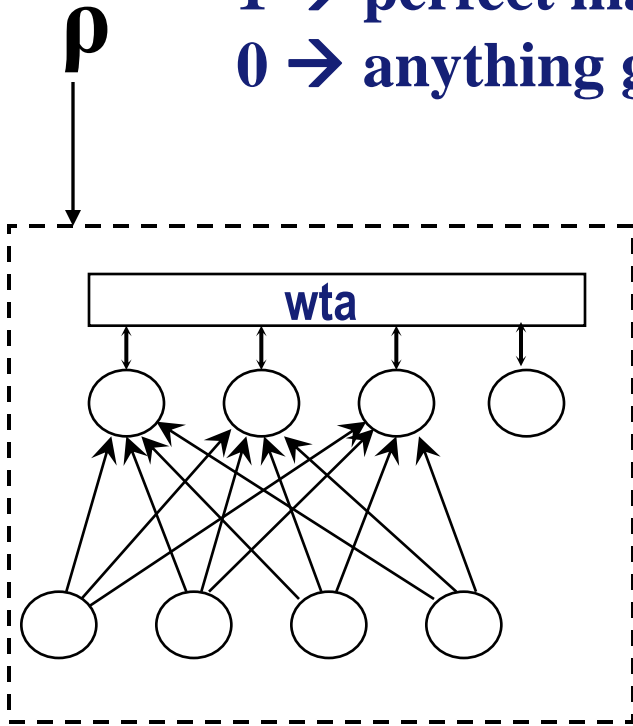
$\rho = 0.99$: A ?



$\rho = 0.2$

Generalization with vigilance, ρ

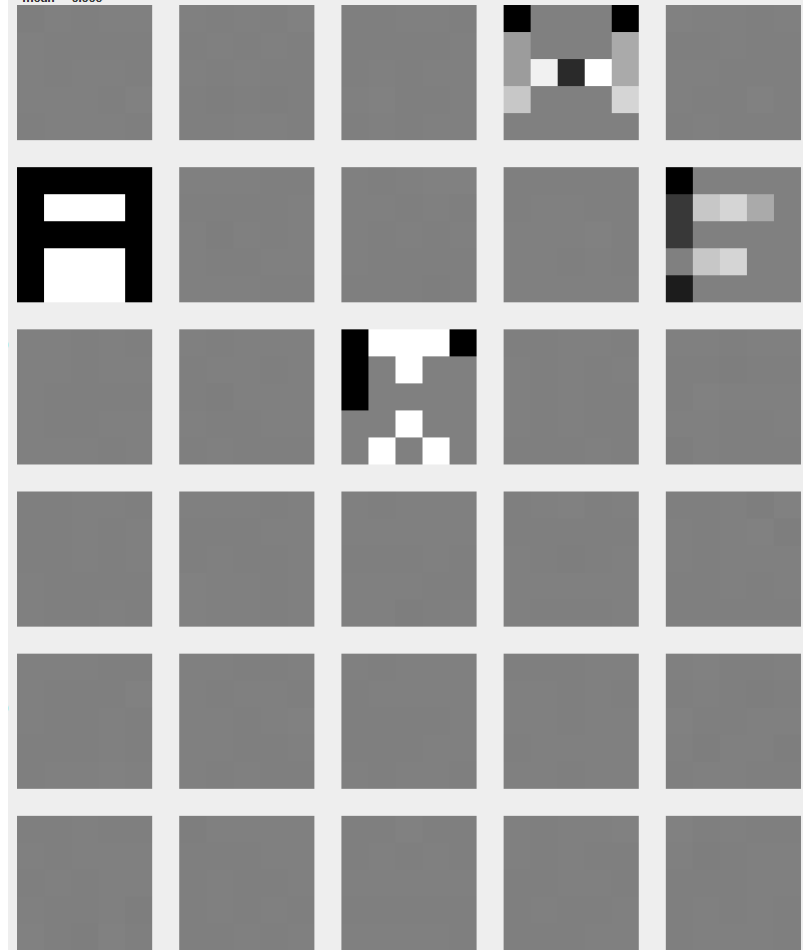
1 \rightarrow perfect match
0 \rightarrow anything goes



$\rho = 0.2$: A B C ... X Y Z

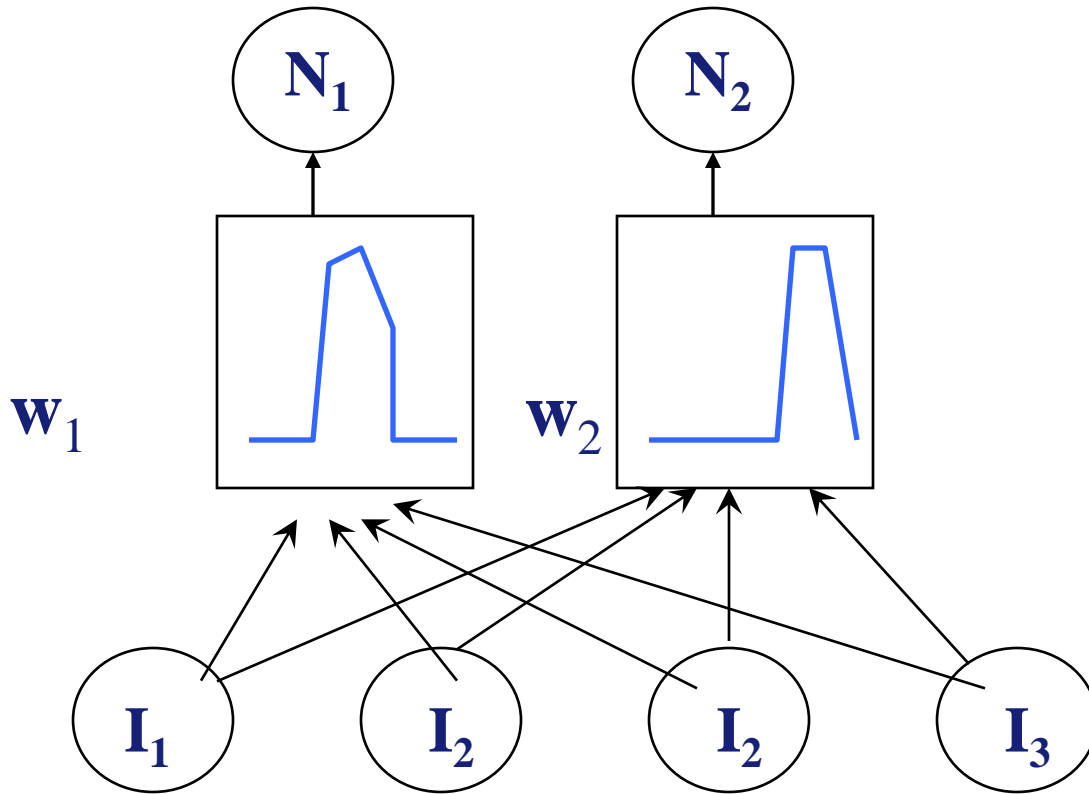
$\rho = 0.99$: A

Learns it!



$\rho = 0.2$

ART in hardware? 4 challenges



Winner =

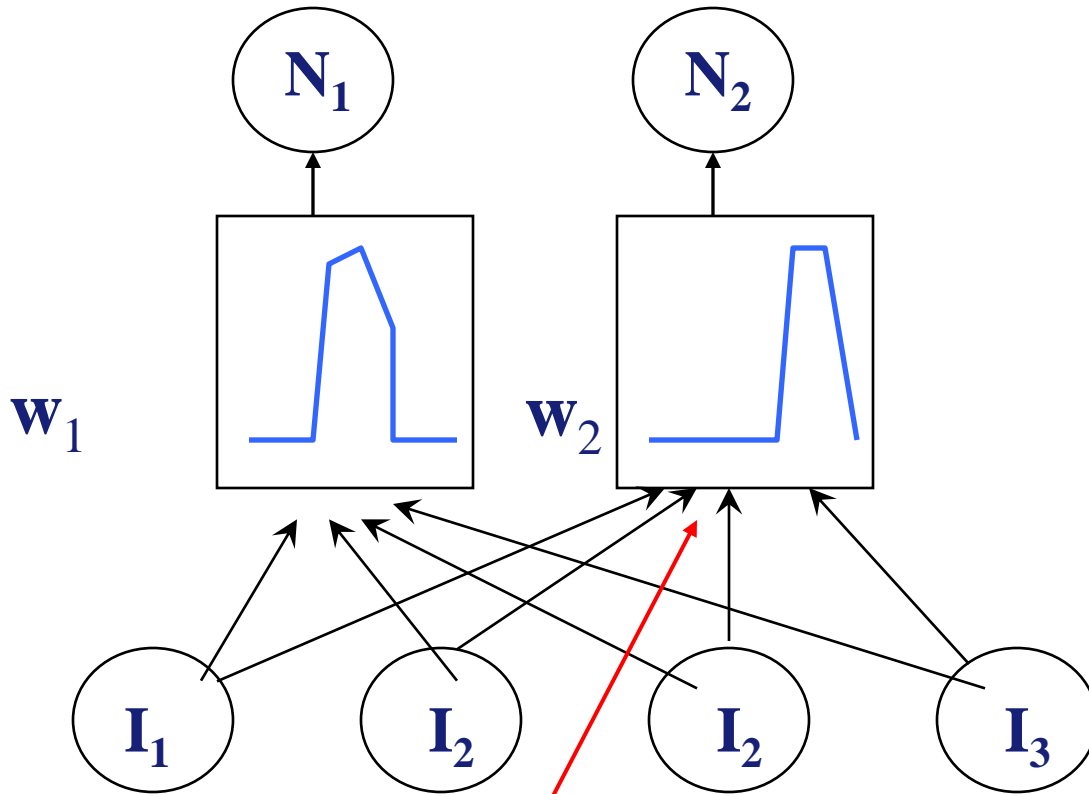
$$\max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

ART in hardware? 4 challenges

1. Nonregular



2. Dynamic allocation

3. Bounded resource

Winner =

$$\max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

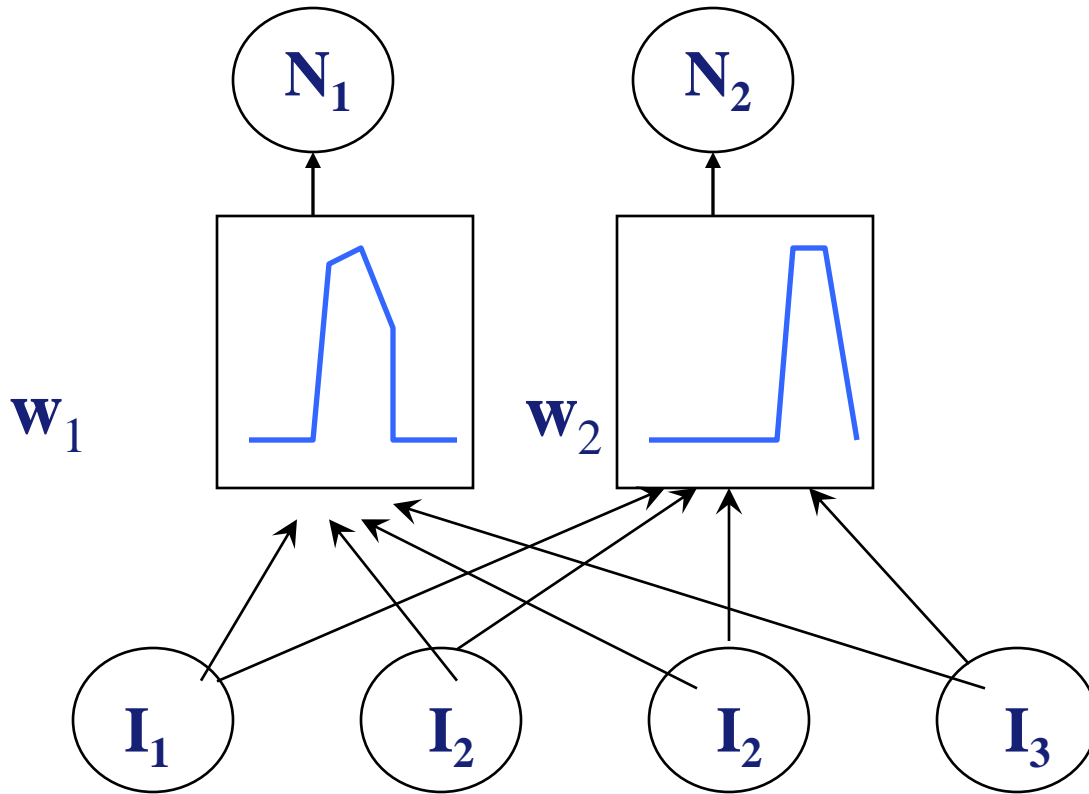
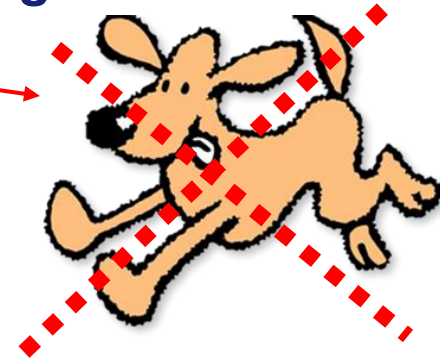
where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

4. Nonlocal

ART in hardware? 4 challenges

1. Nonregular



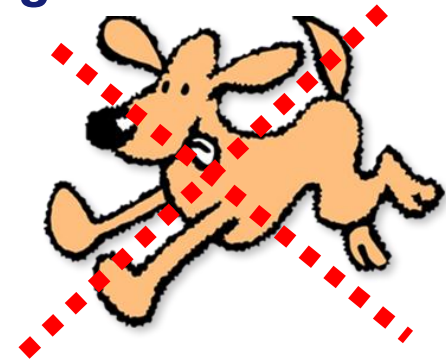
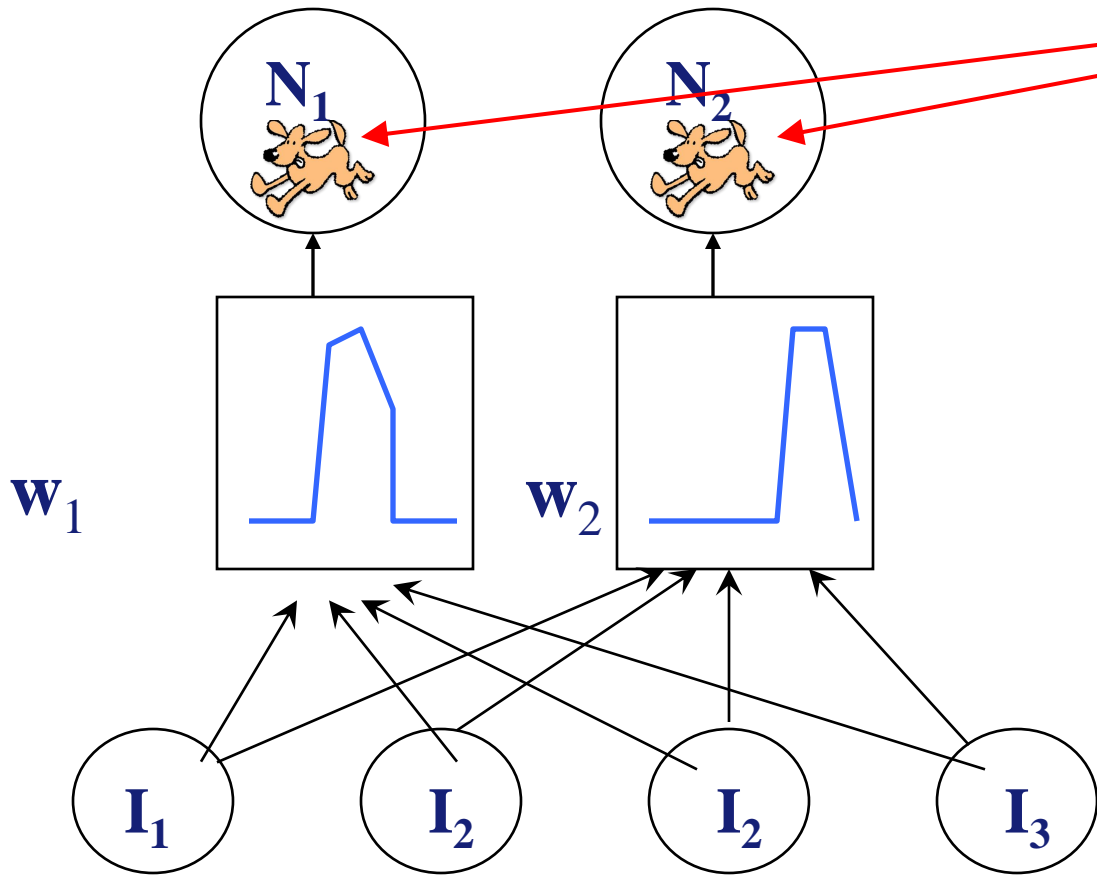
Winner =

$$\max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

ART in hardware? 4 challenges



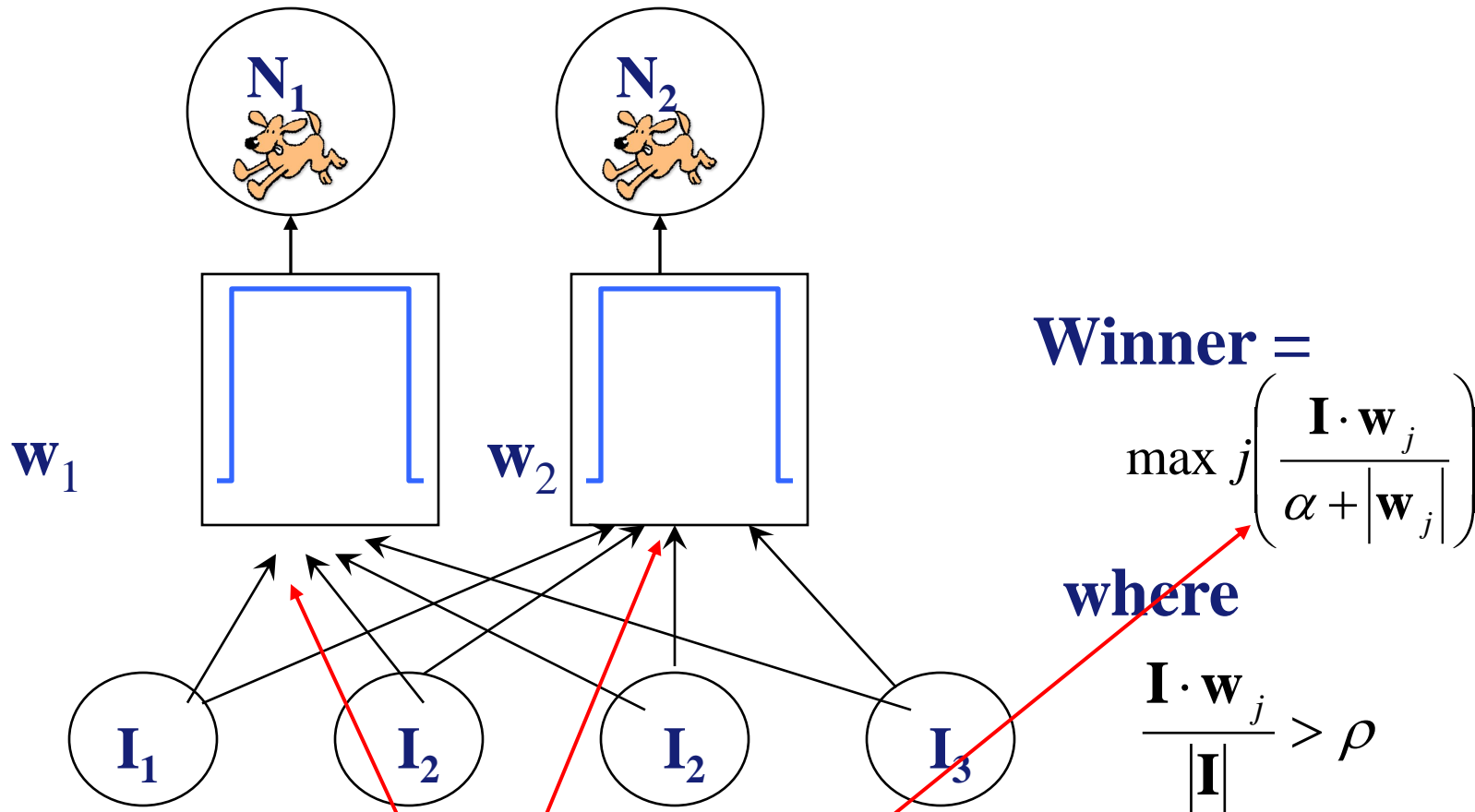
Winner =

$$\max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

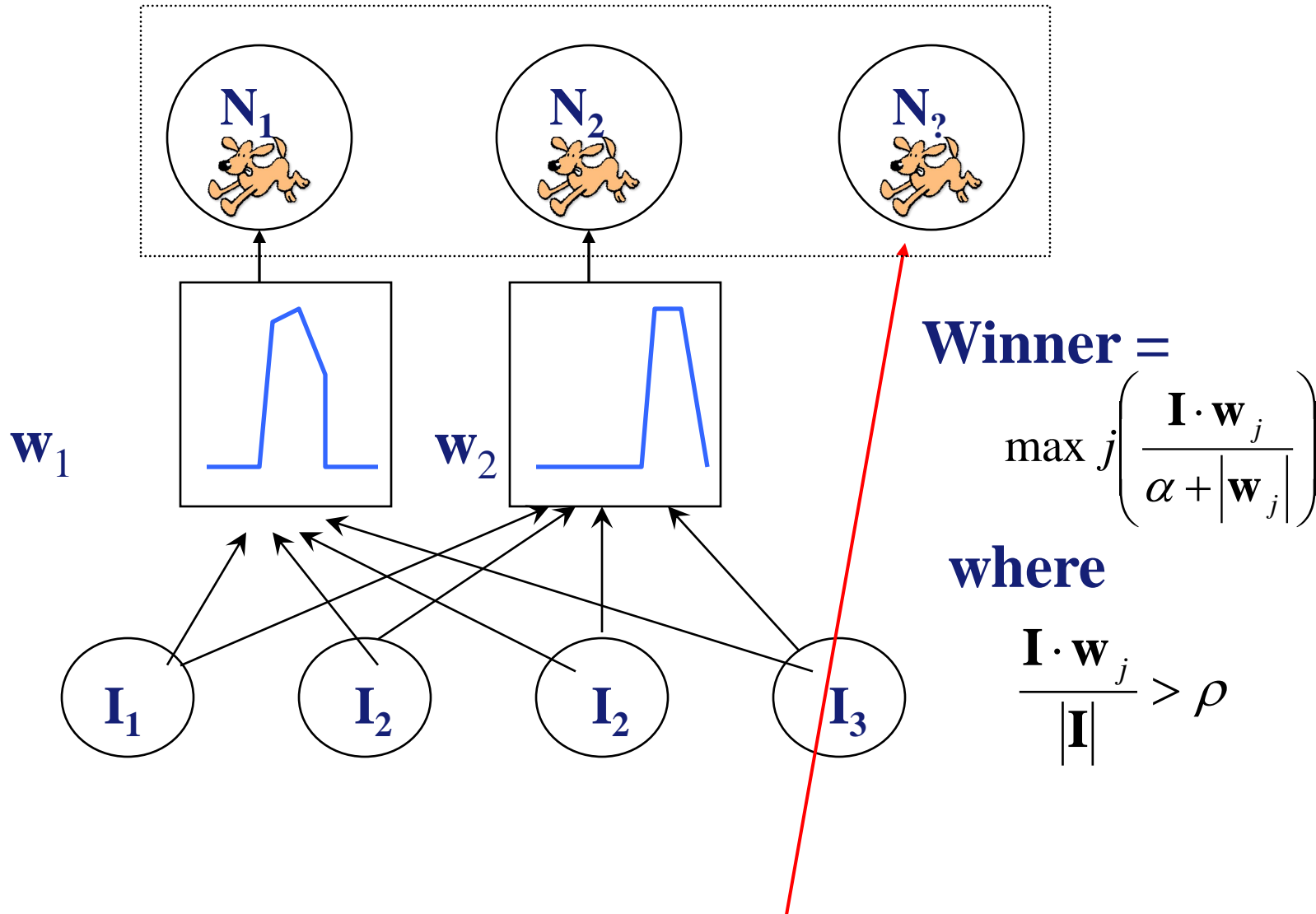
ART in hardware? 4 challenges



2. (Dynamic allocation)

Preallocated filters—poor fit to everything, but will resonate if necessary

ART in hardware? 4 challenges



Winner =

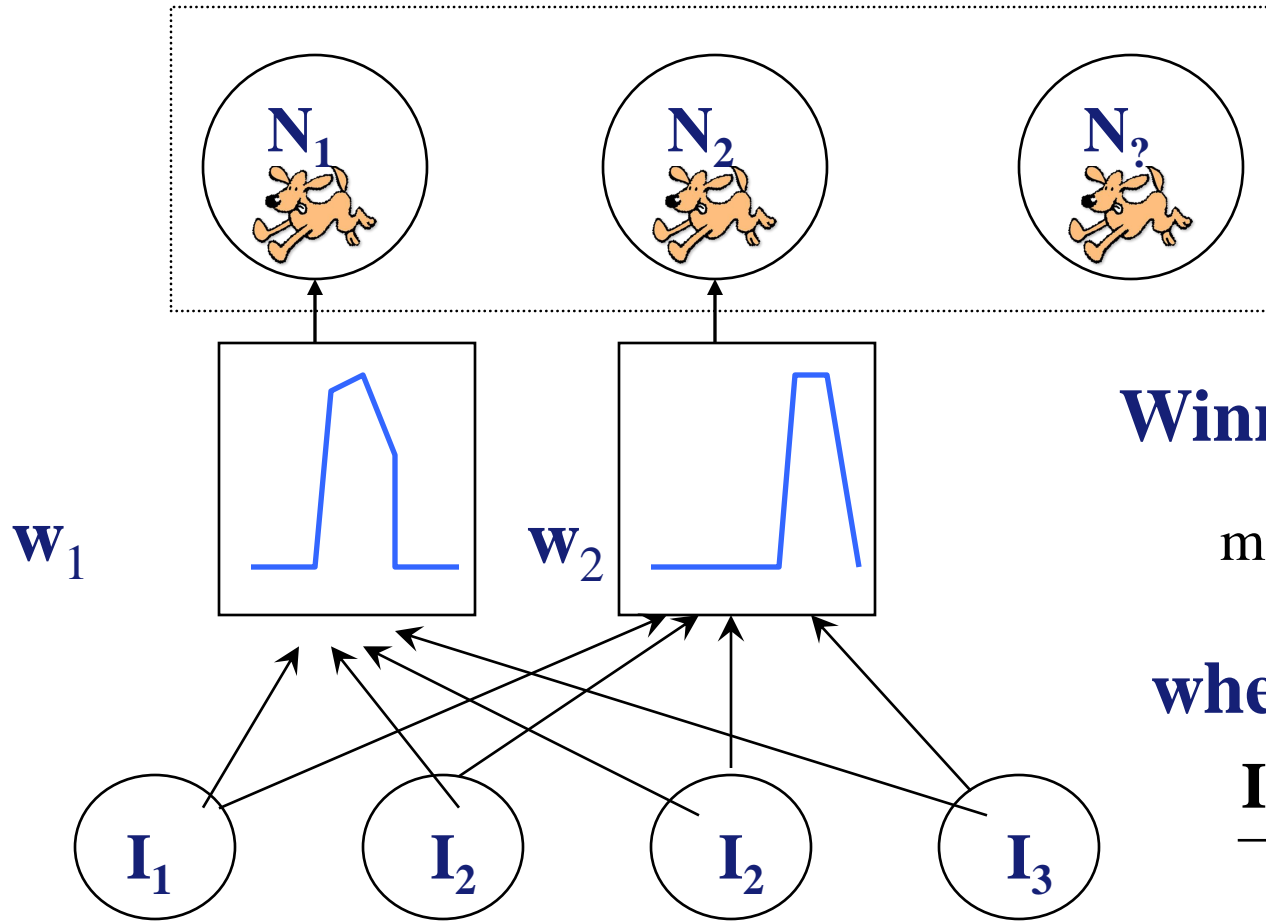
$$\max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

3. (Bounded resource) "I don't know" neuron. If all neurons have learned and novel pattern comes in, the "I don't know" neuron wins.

ART in hardware? 4 challenges



Winner =

$$\max_j \left(\frac{\mathbf{I} \cdot \mathbf{w}_j}{\alpha + |\mathbf{w}_j|} \right)$$

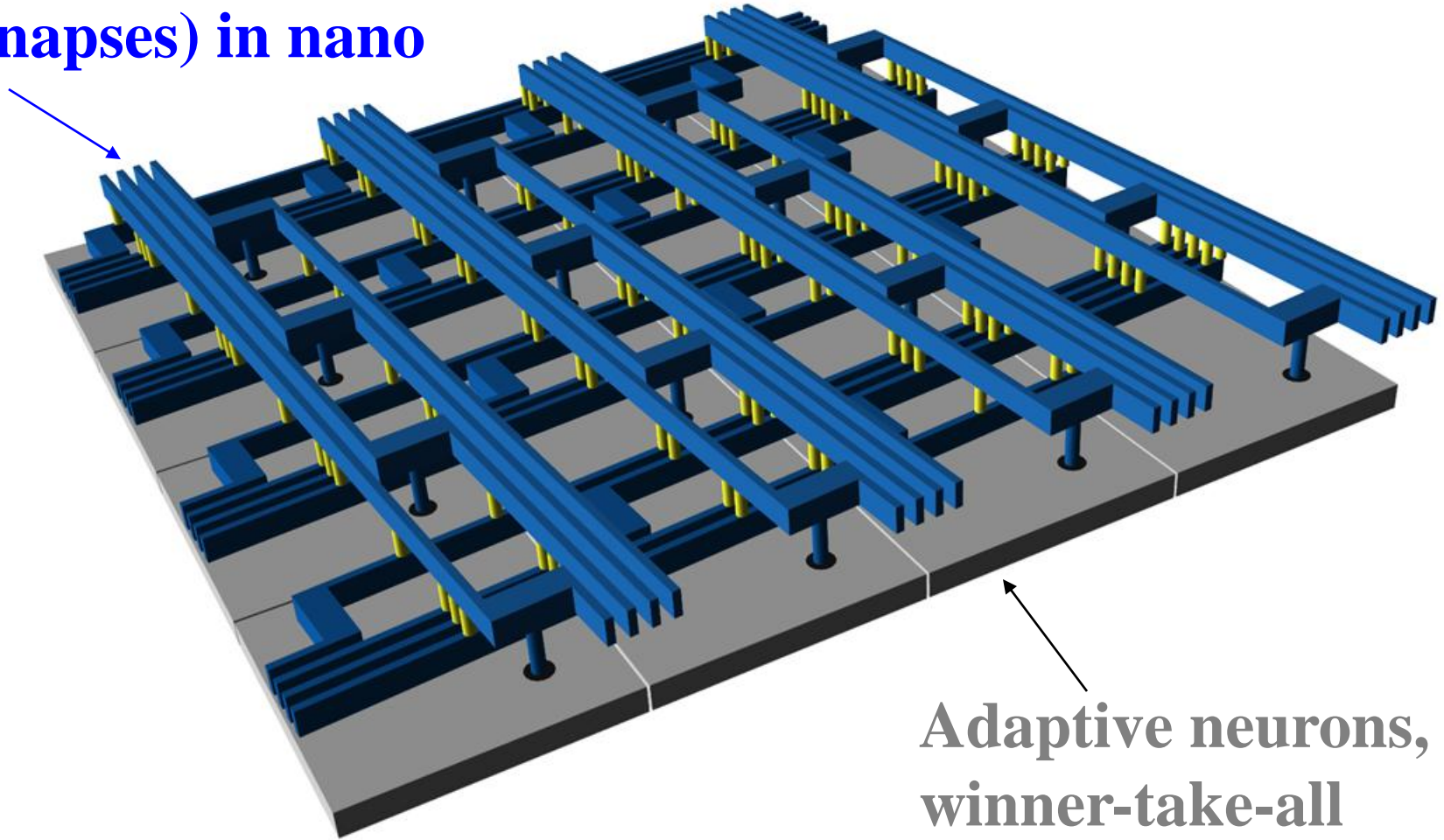
where

$$\frac{\mathbf{I} \cdot \mathbf{w}_j}{|\mathbf{I}|} > \rho$$

**4. (Nonlocal)
Adaptive neurons
(synaptic scaling)**

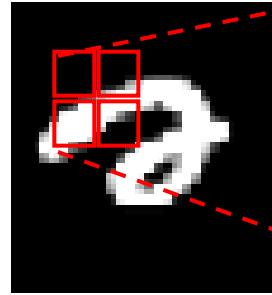
Nano/CMOS layout, mART-1

**Adaptive filters
(synapses) in nano**



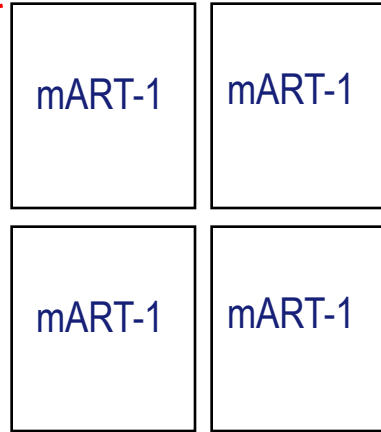
**Adaptive neurons,
winner-take-all
circuits in CMOS**

Fast, incremental learning



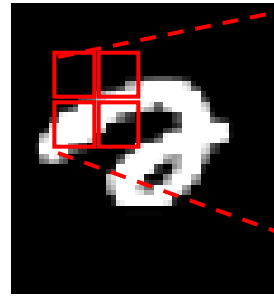
MNIST digits

center-surround
processing



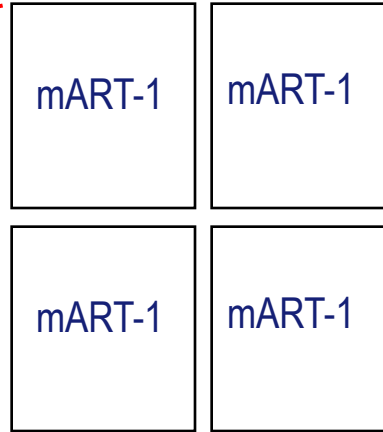
Low vigilance

Fast, incremental learning

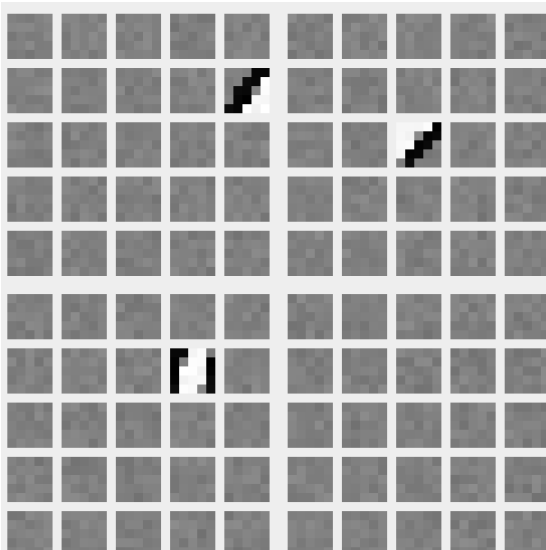


MNIST digits

center-surround processing

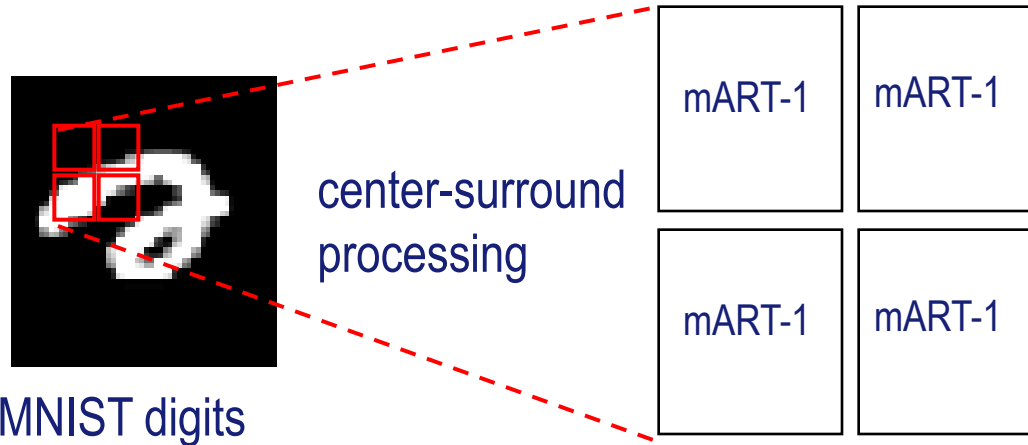


Low vigilance

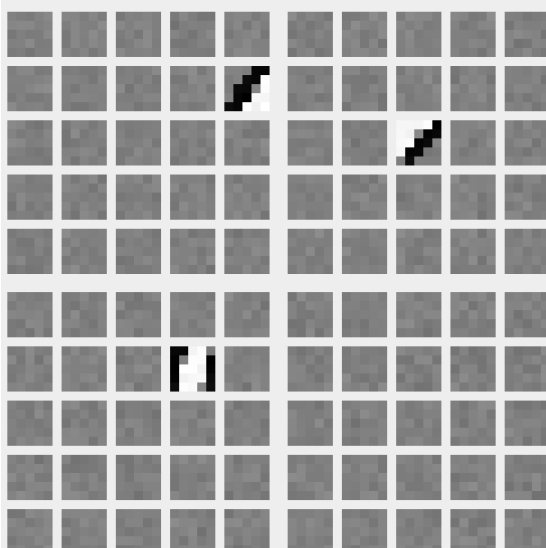


After 1 digit

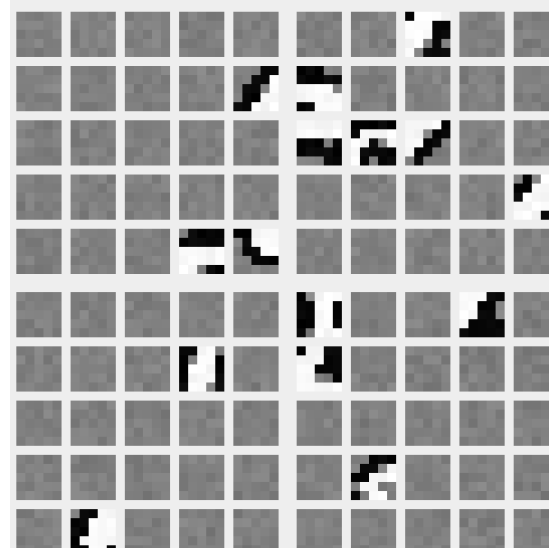
Fast, incremental learning



Low vigilance

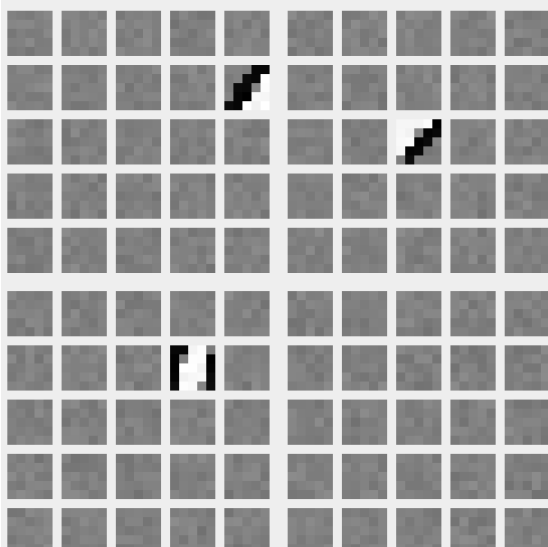
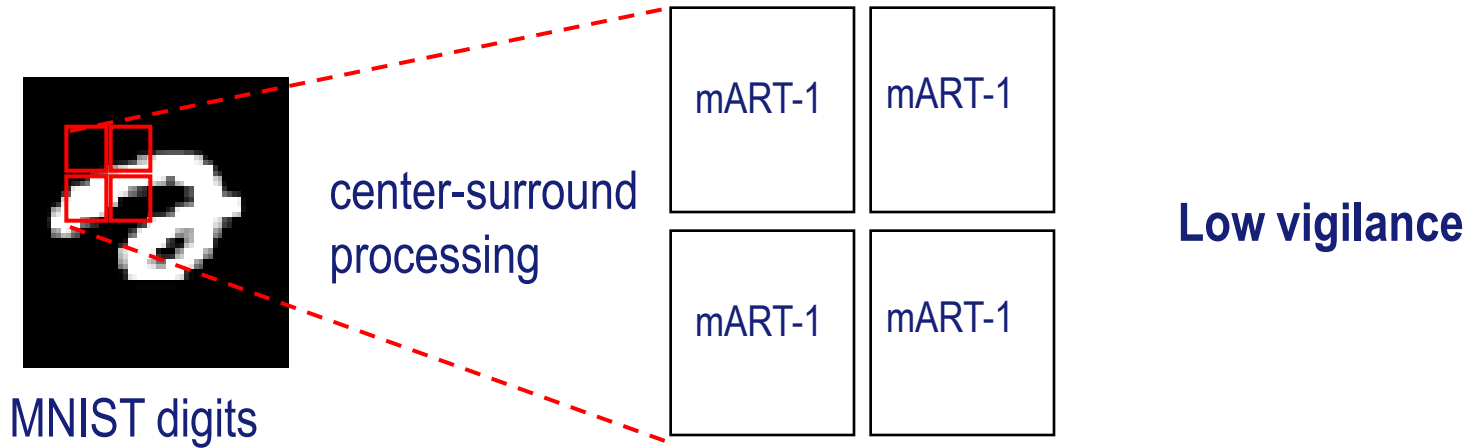


After 1 digit

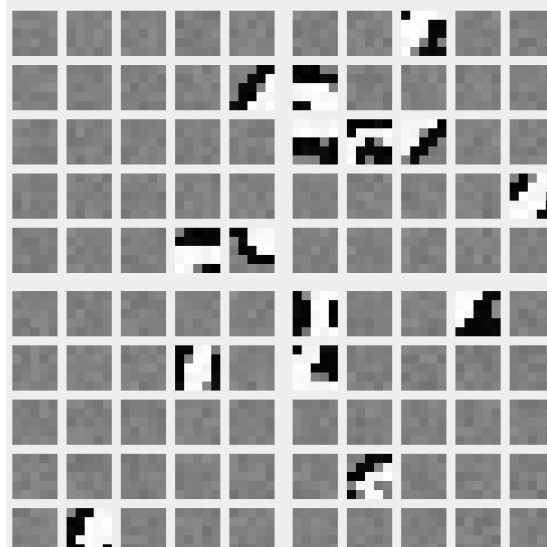


After 30 digits

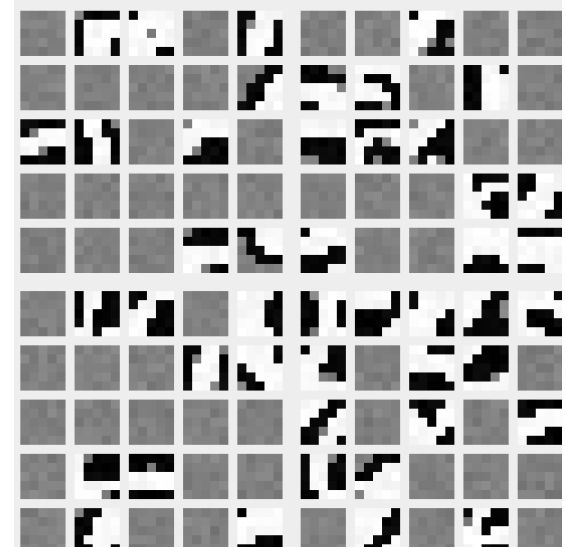
Fast, incremental learning



After 1 digit



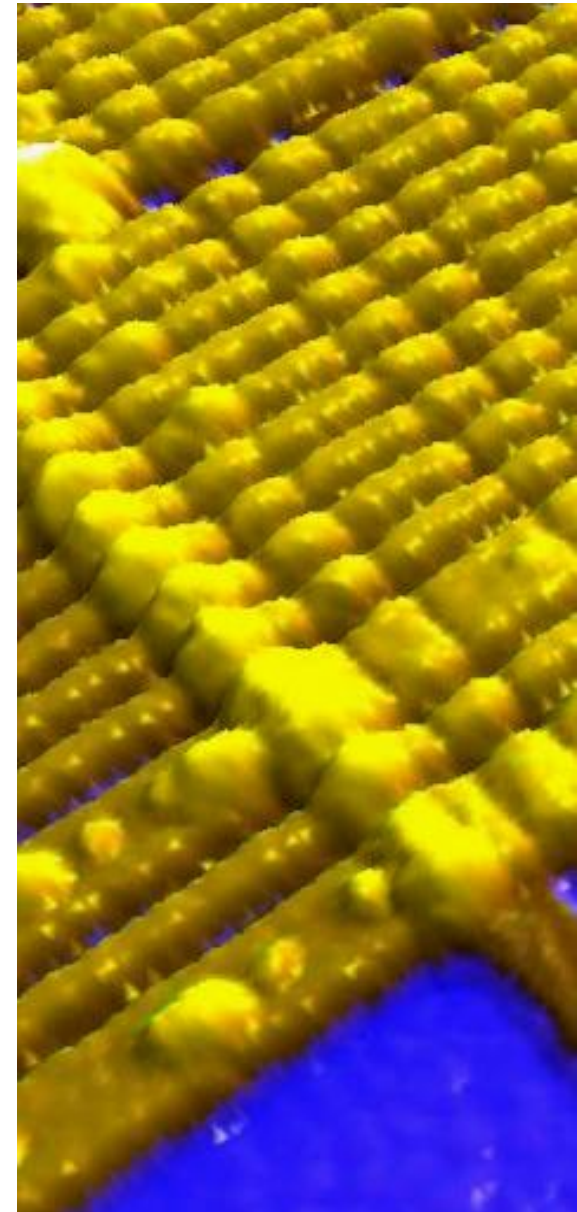
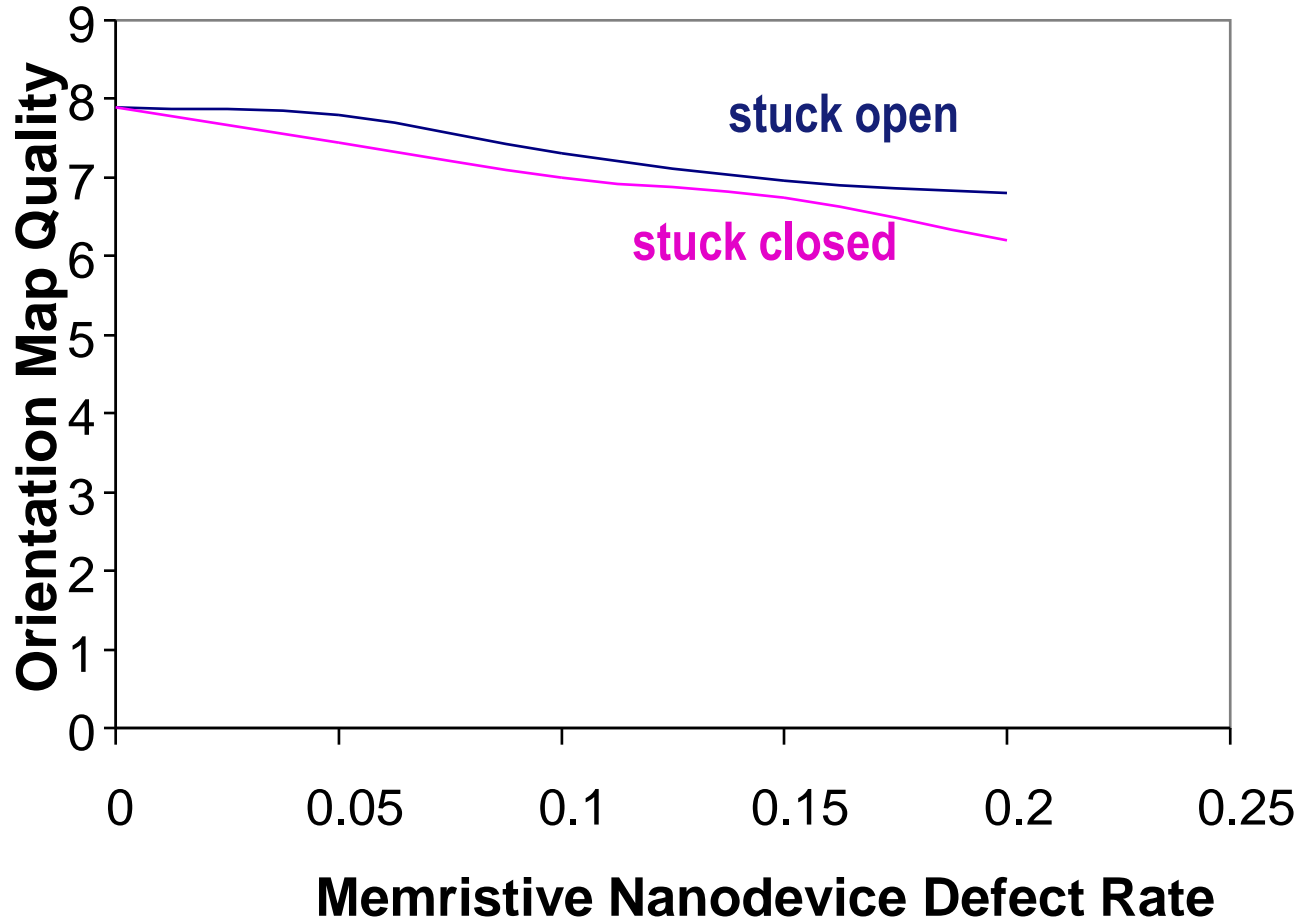
After 30 digits



After 60,000 digits

Defect, device-variation strategy: *Ignore them*

Defect Tolerance of Self-Organized Orientation Map



Summary

Memristive nanodevices → synapses

ART networks → regular structure (CMOS + nano)

Fast or slow learning

Tolerant of defects and device variation