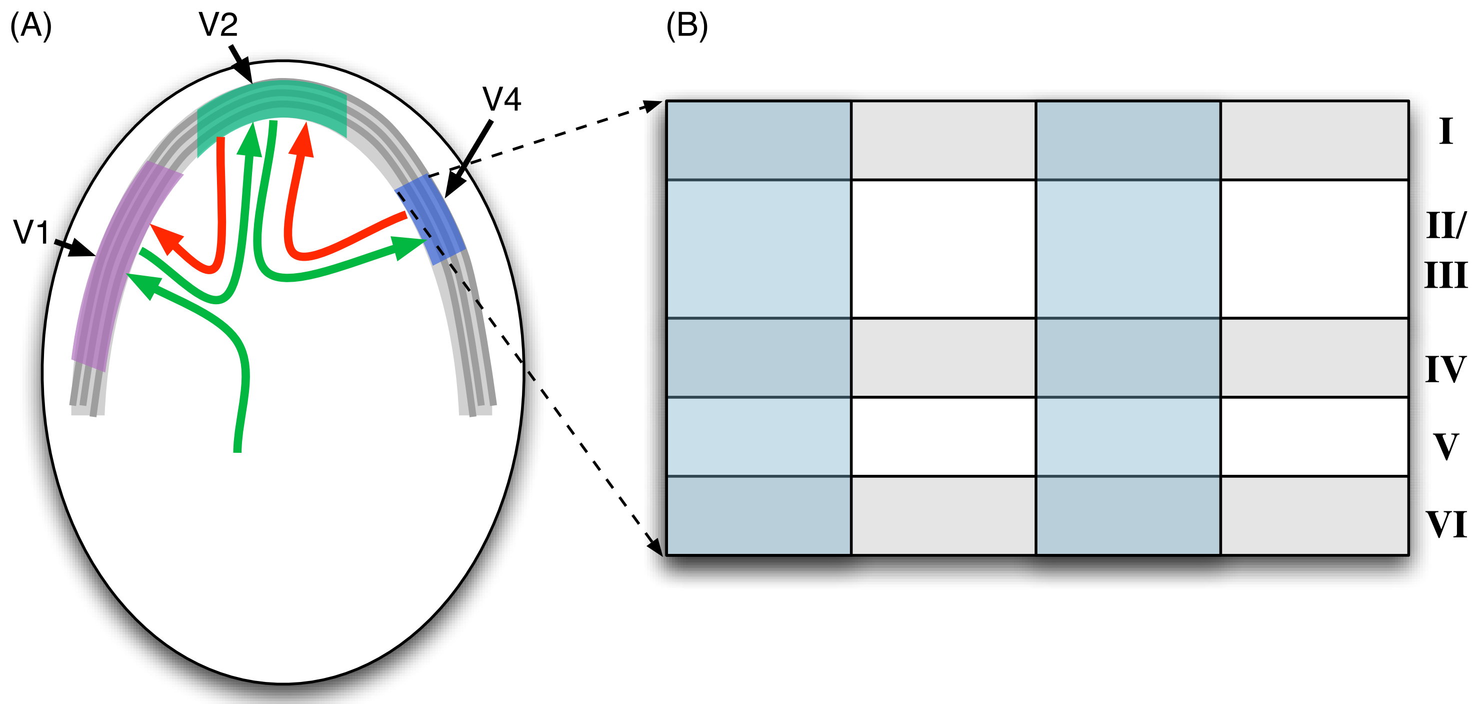
Numenta®

A Mathematical Model for Cortical Circuits

Dileep George

Neocortex Organization : 2 minute version



Cortical Hierarchy

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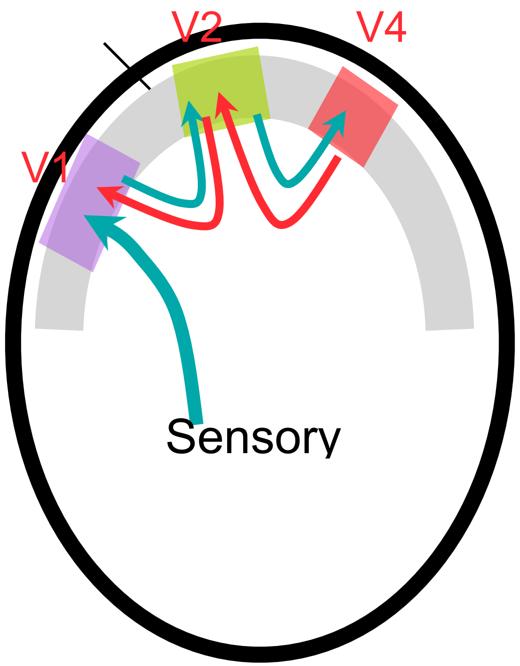

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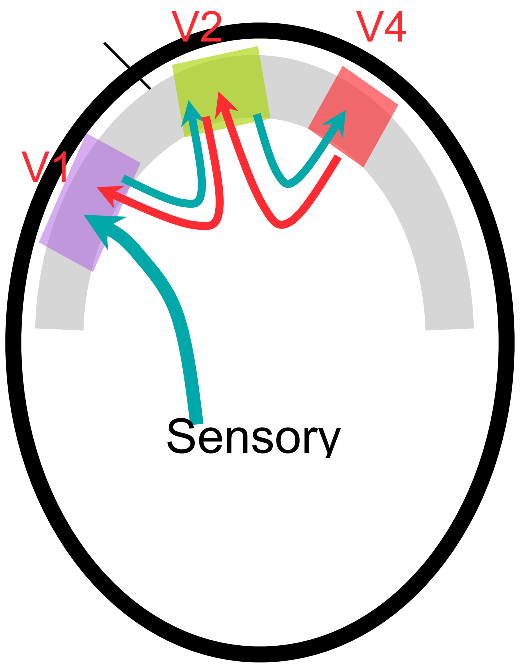

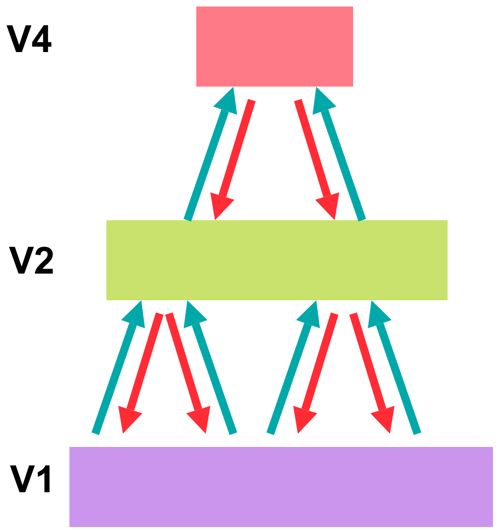

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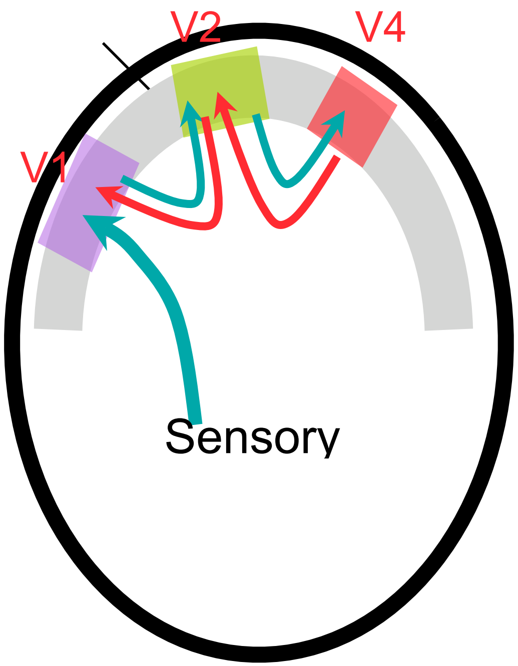

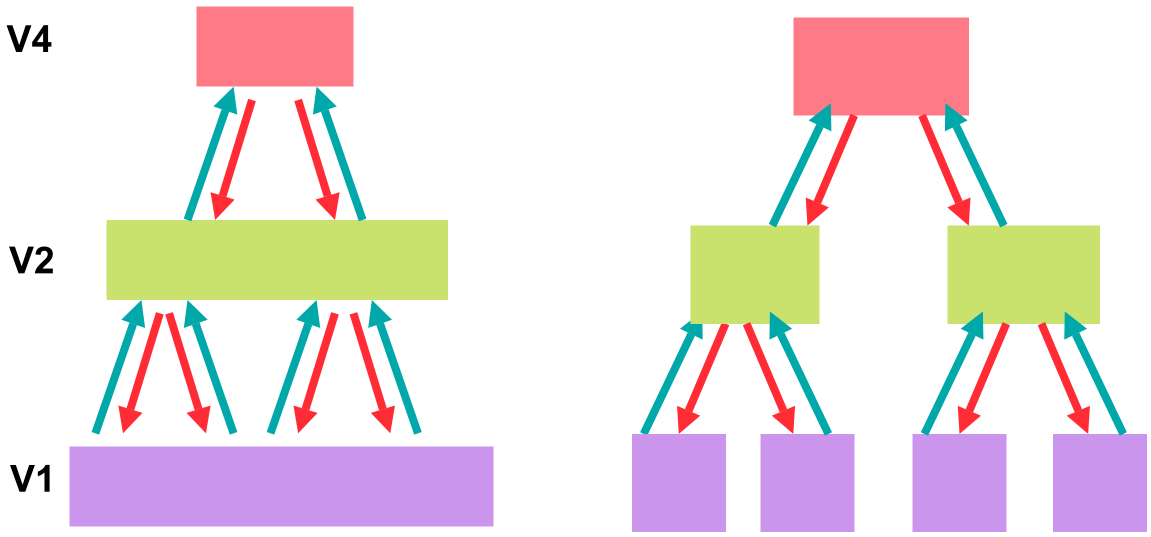

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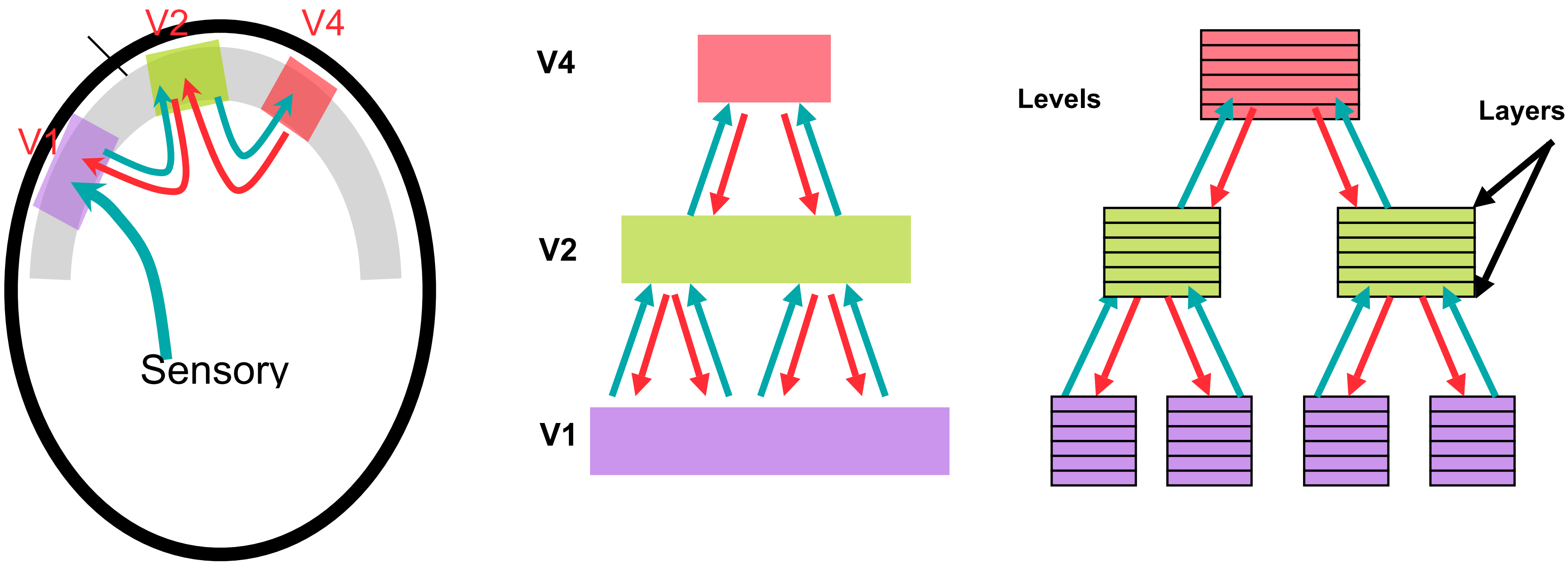

Hardware/software design challenges

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- **Adaptive computing**
  - Reconfigures connections.
  - Might need to allocate more resources during the initial stages of learning

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- **Solution?**
  - Abstract and formalize
  - Parameterize the degrees of freedom

Aspiration

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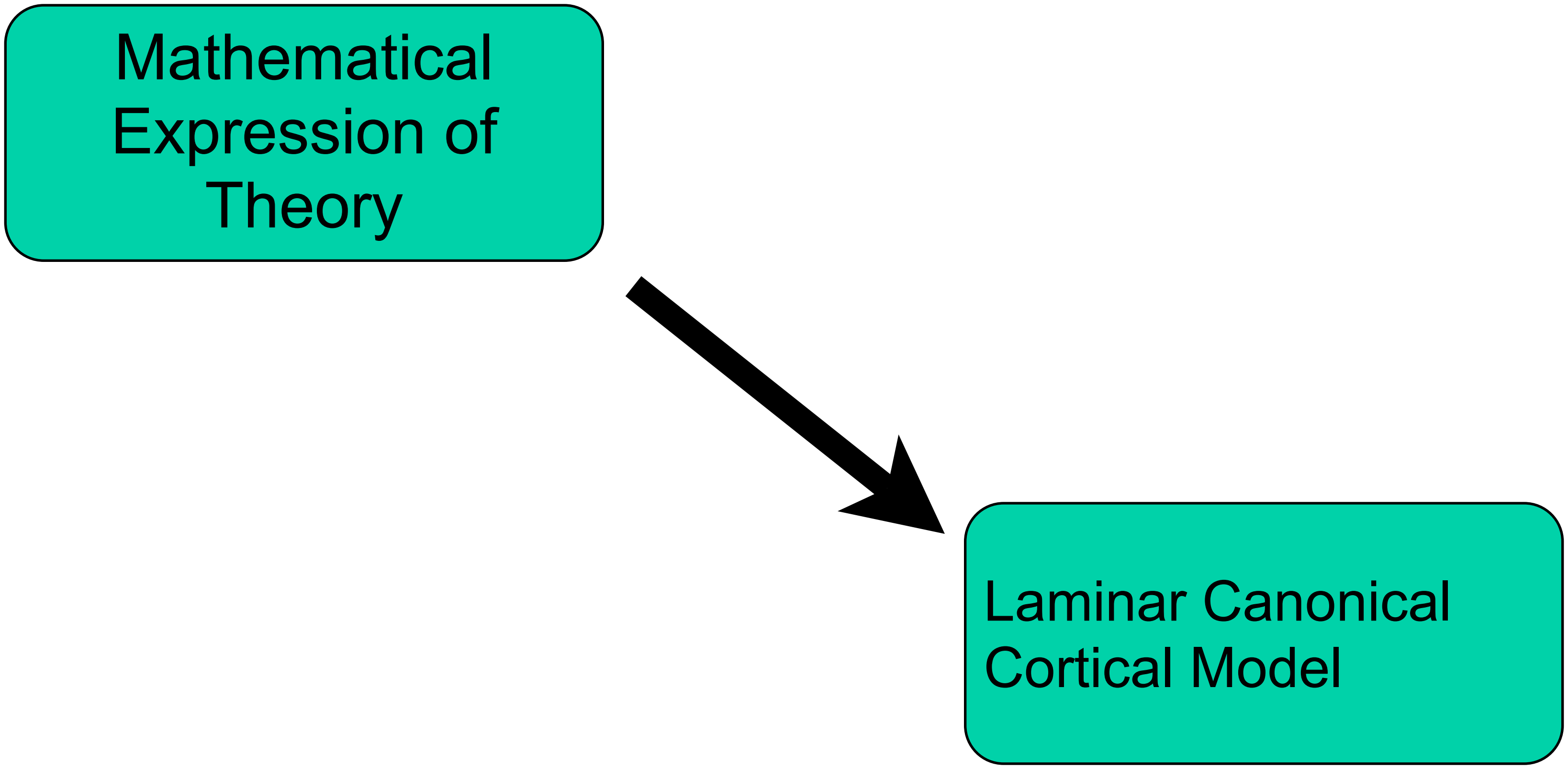
- Audacious
  - But this is the season of Hope and Change
  - Yes we can!!

Agenda

Mathematical Expression of Theory

Agenda


Mathematical Expression of Theory
Laminar Canonical Cortical Model

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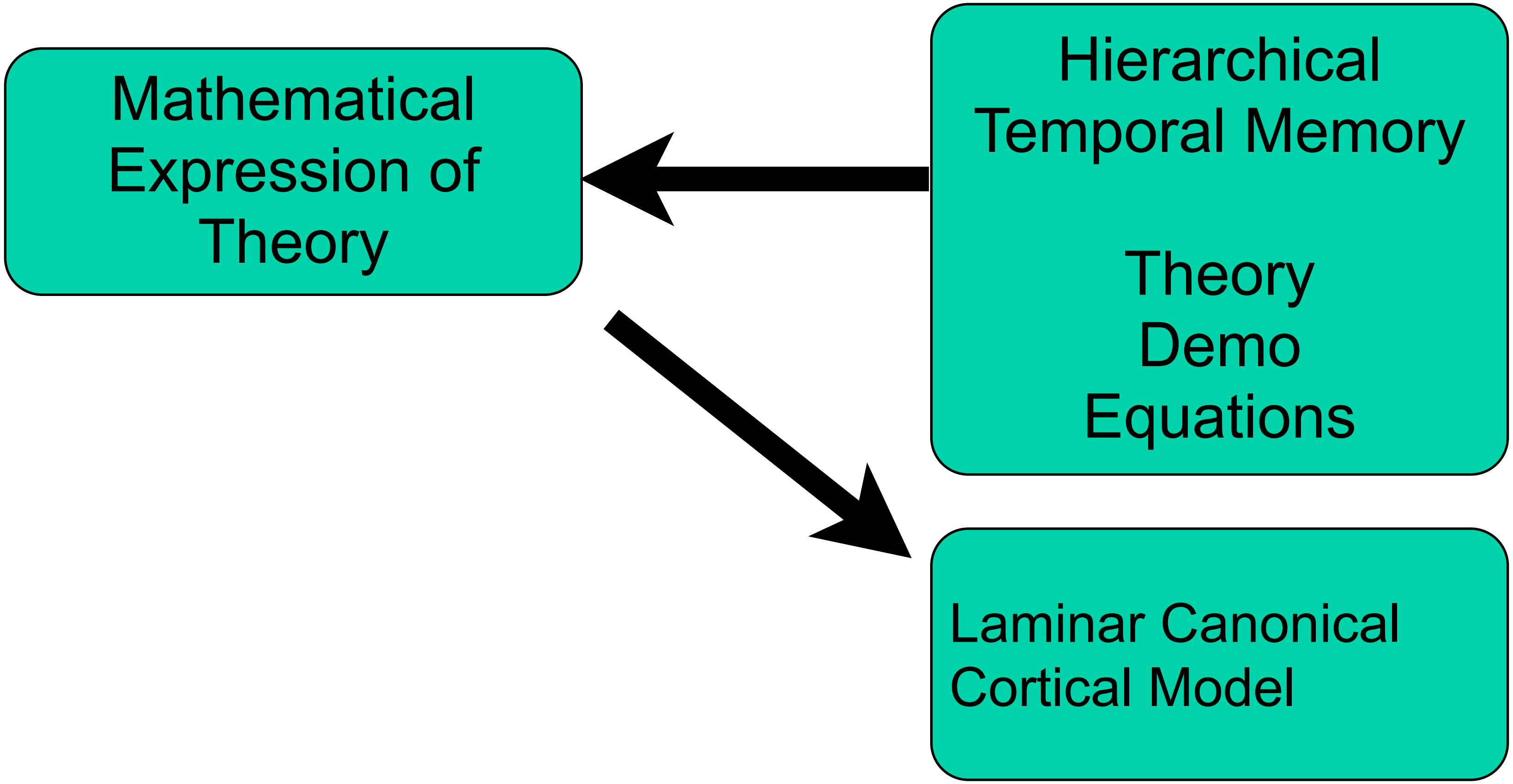
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Hierarchical Temporal Memory
Theory
Demo
Equations
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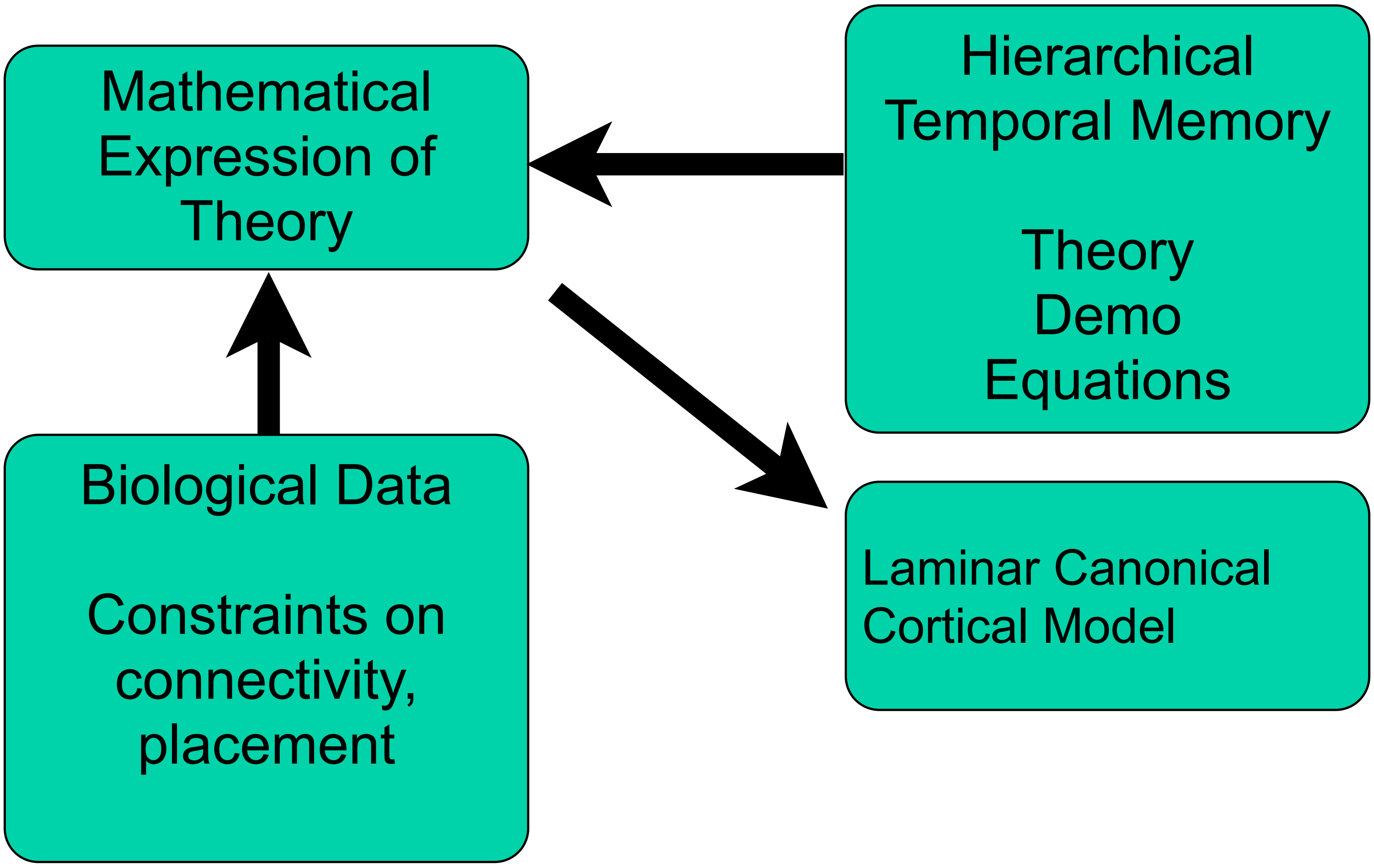
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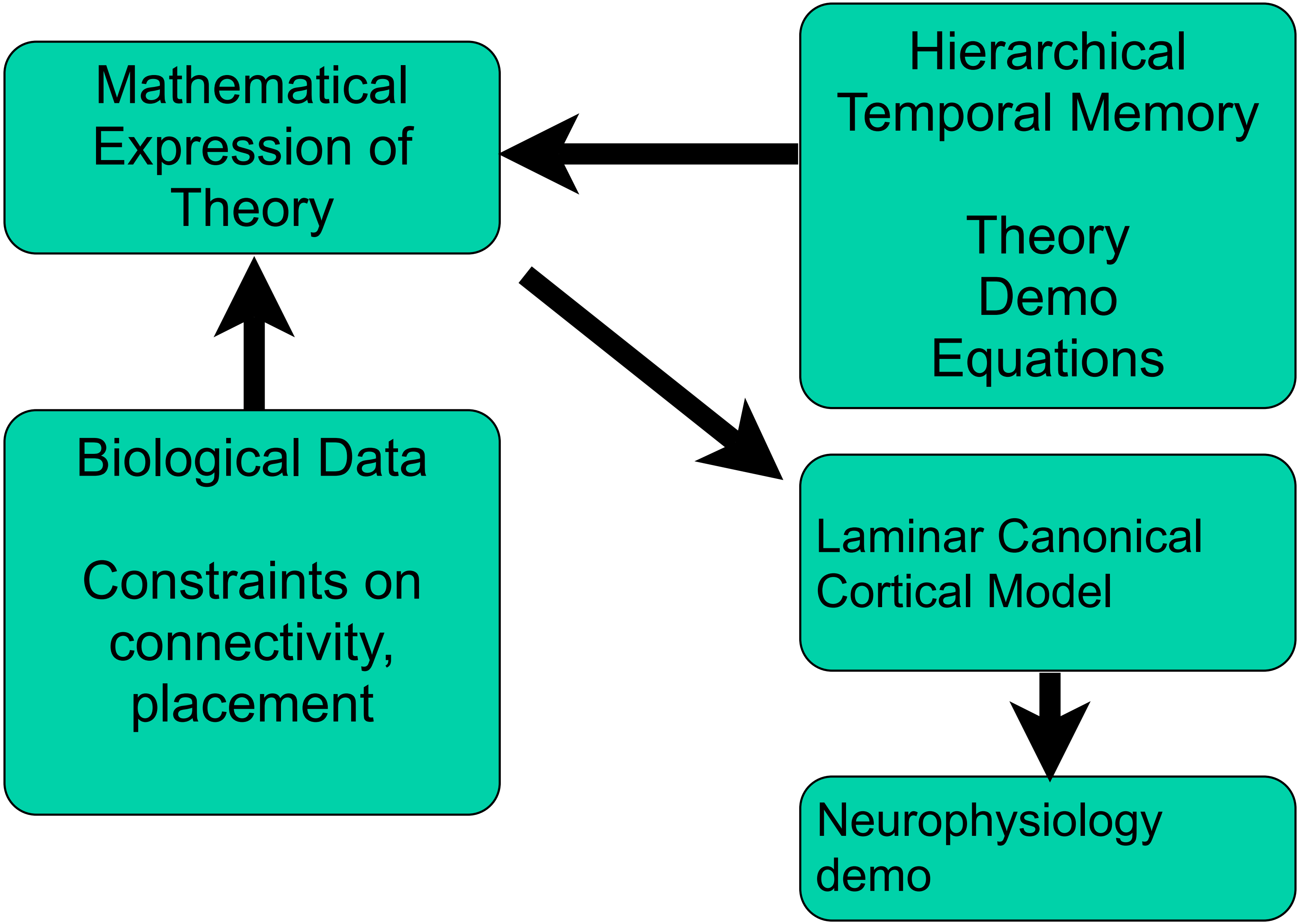
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Neurophysiology demo

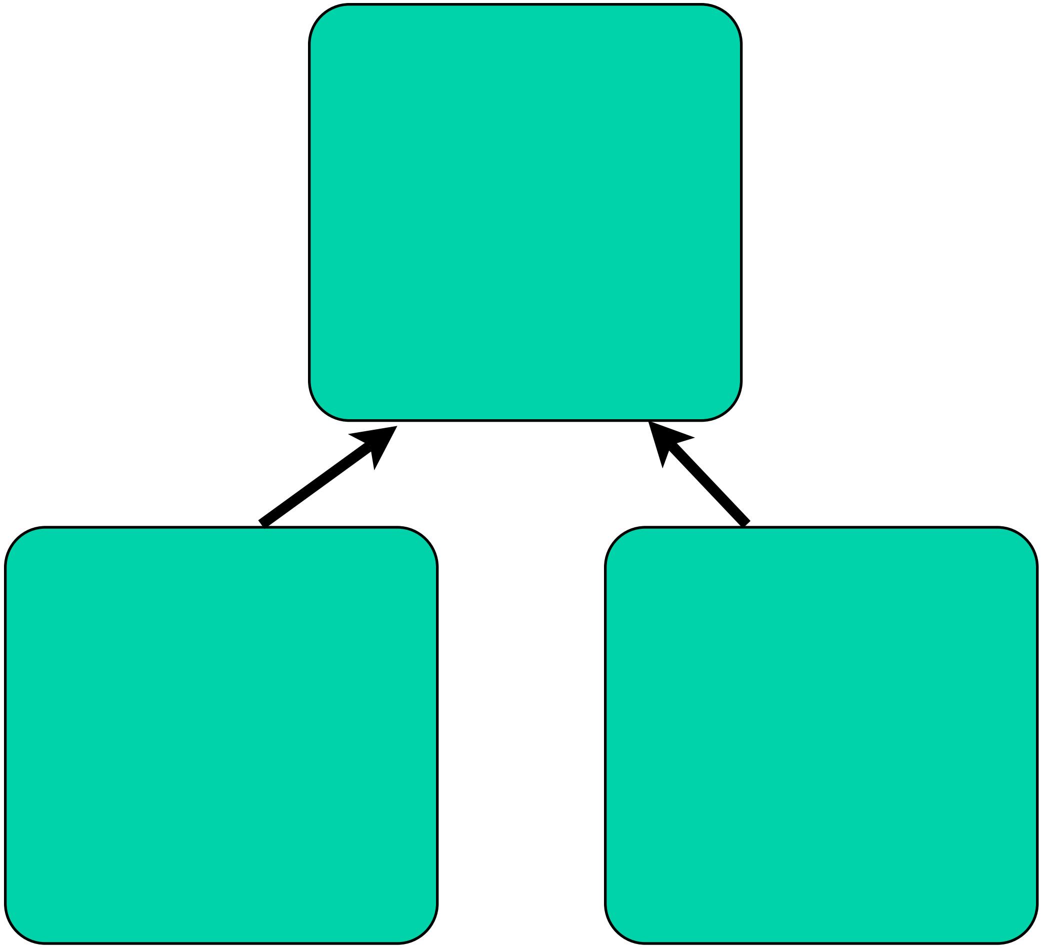
Hierarchical Temporal Memory

- Is a model for neocortical operation
  - Nodes organized in a hierarchy
  - Each node uses the same algorithm

Features of HTM (Principles of Cortex-like computation)

- Hierarchy in space and time
- Feed-forward and feedback connections
- Common cortical algorithm
- Inference using Bayesian belief propagation
- Sparse Distributed Representations
- Prediction using temporal context
- Biologically accurate

Hierarchical Temporal Memory



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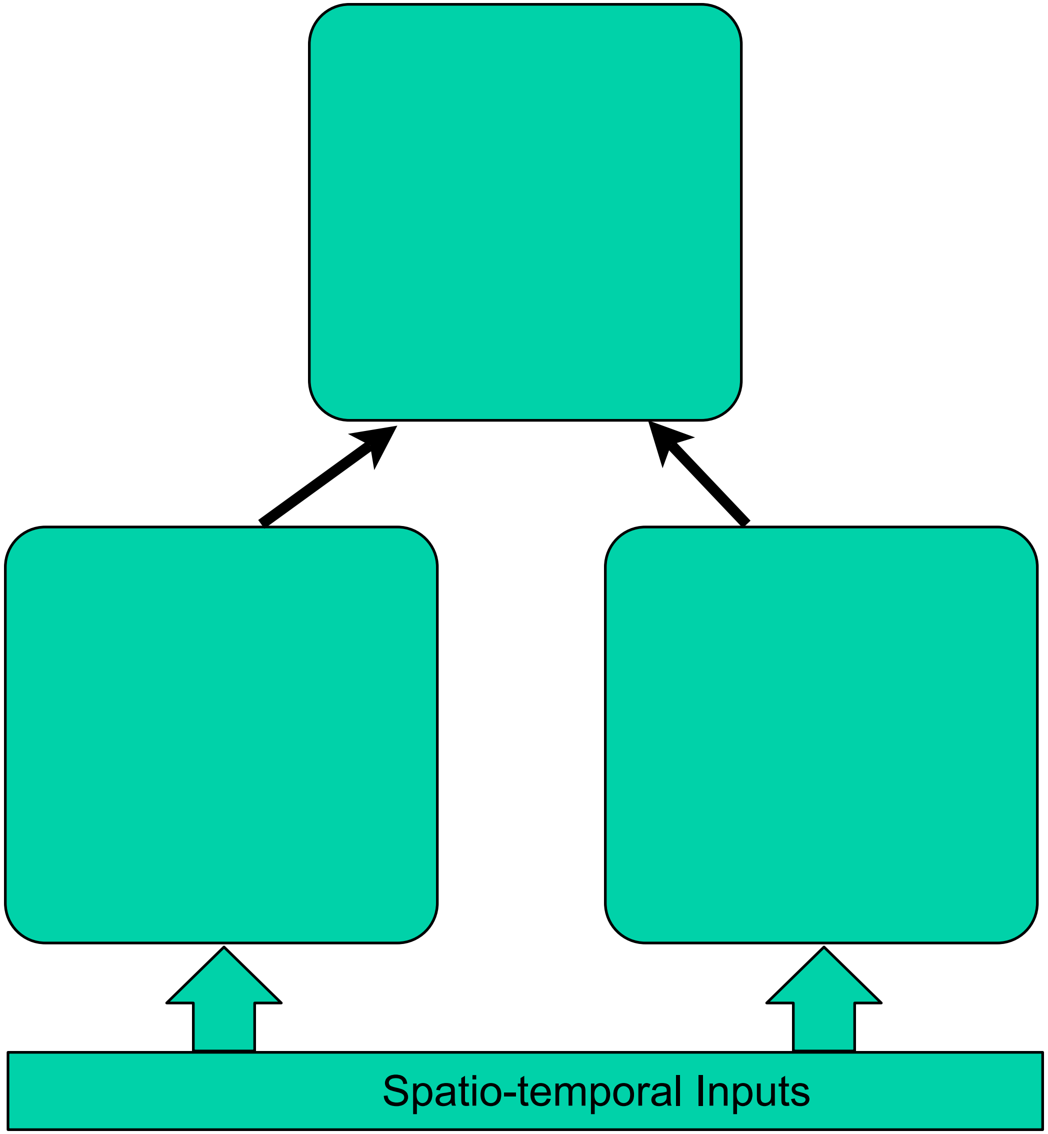

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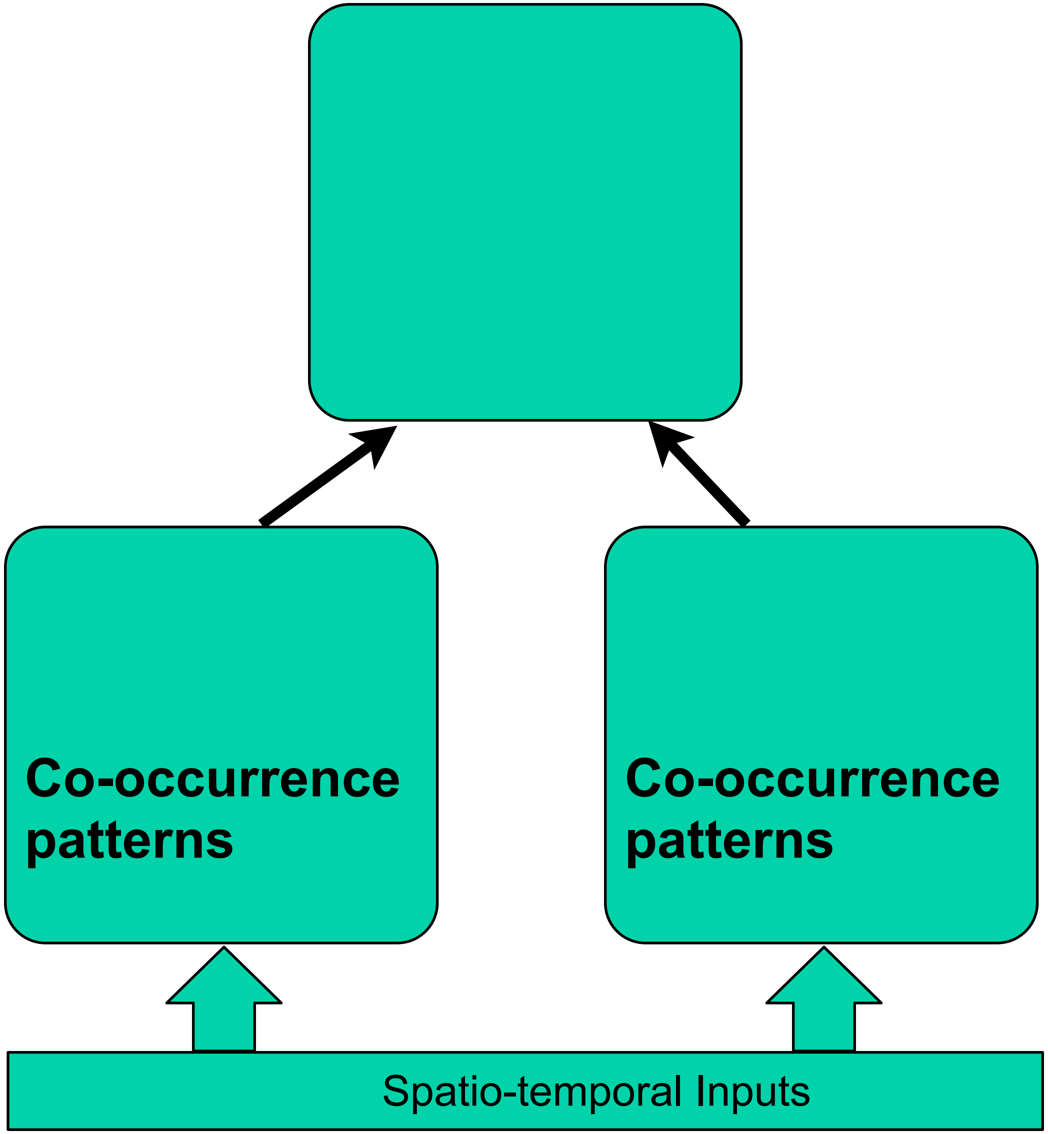

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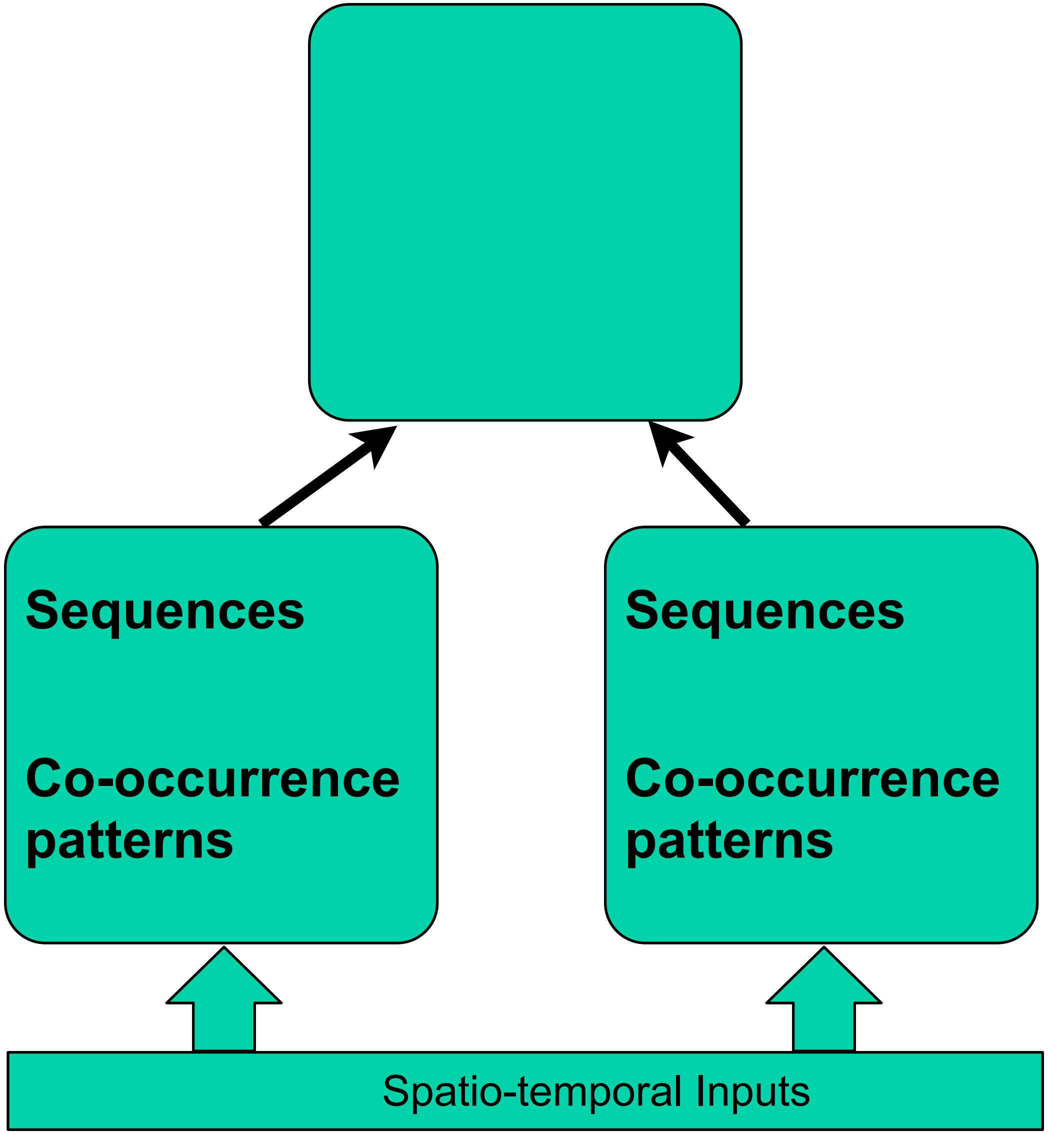

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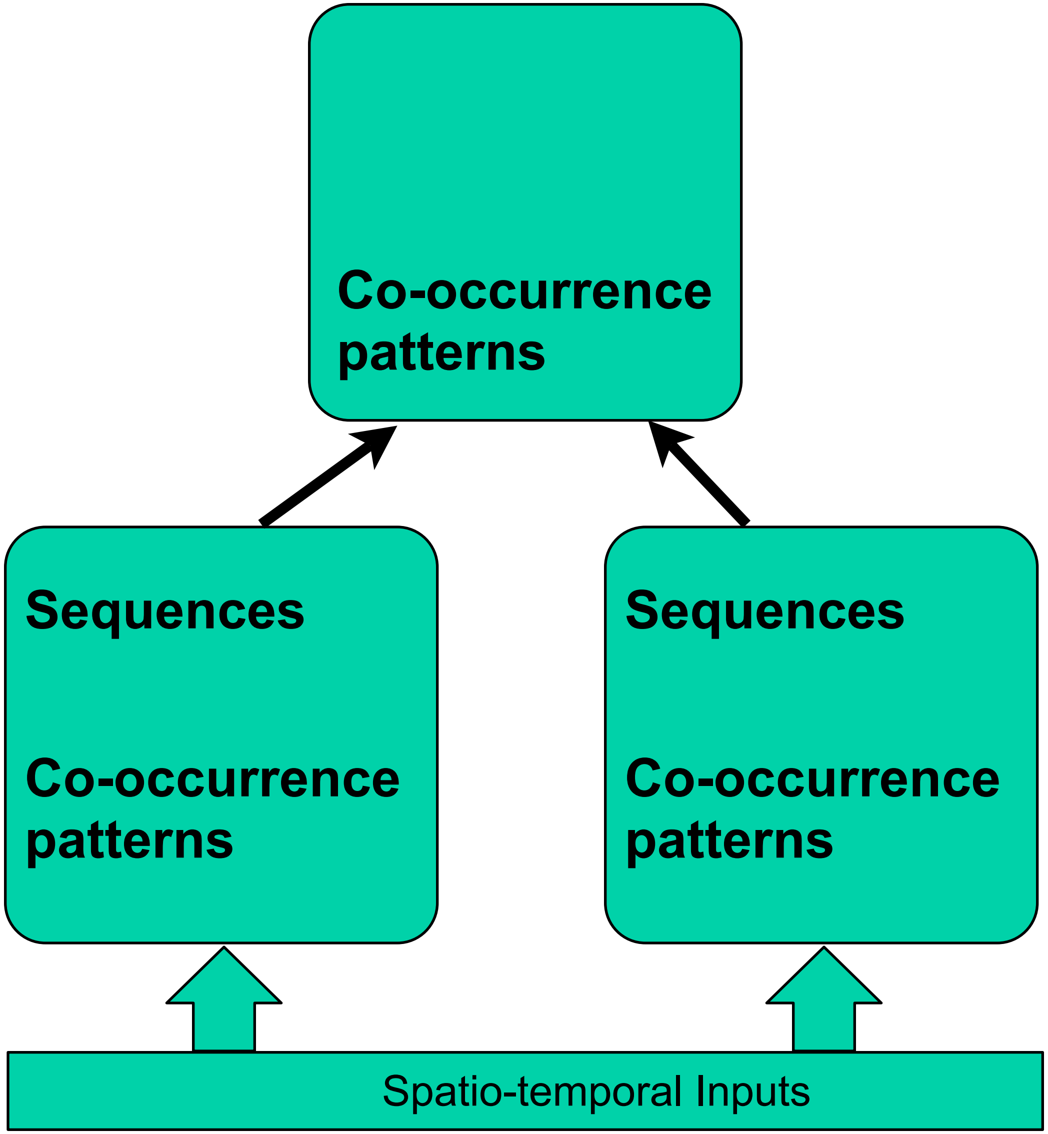

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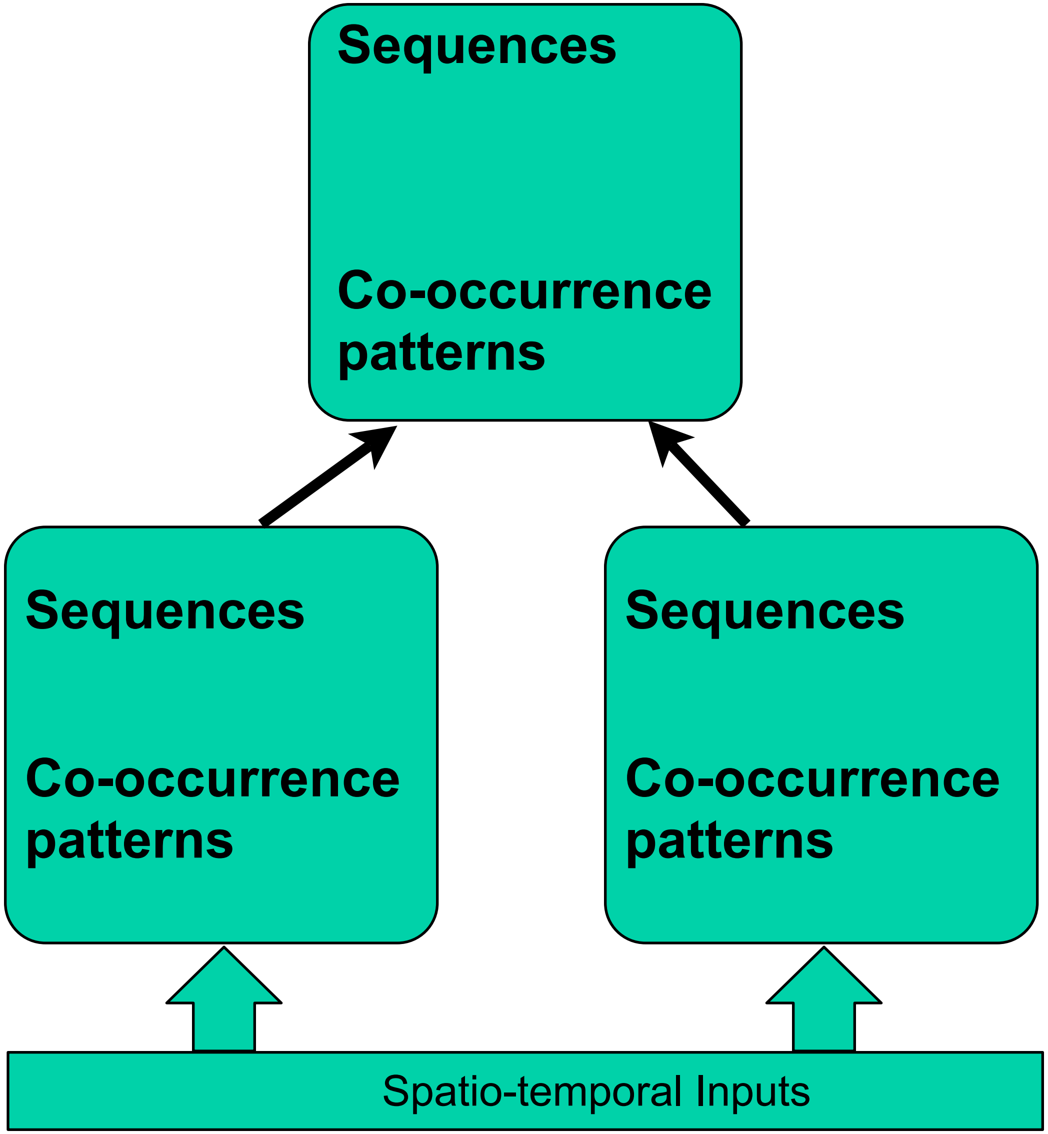
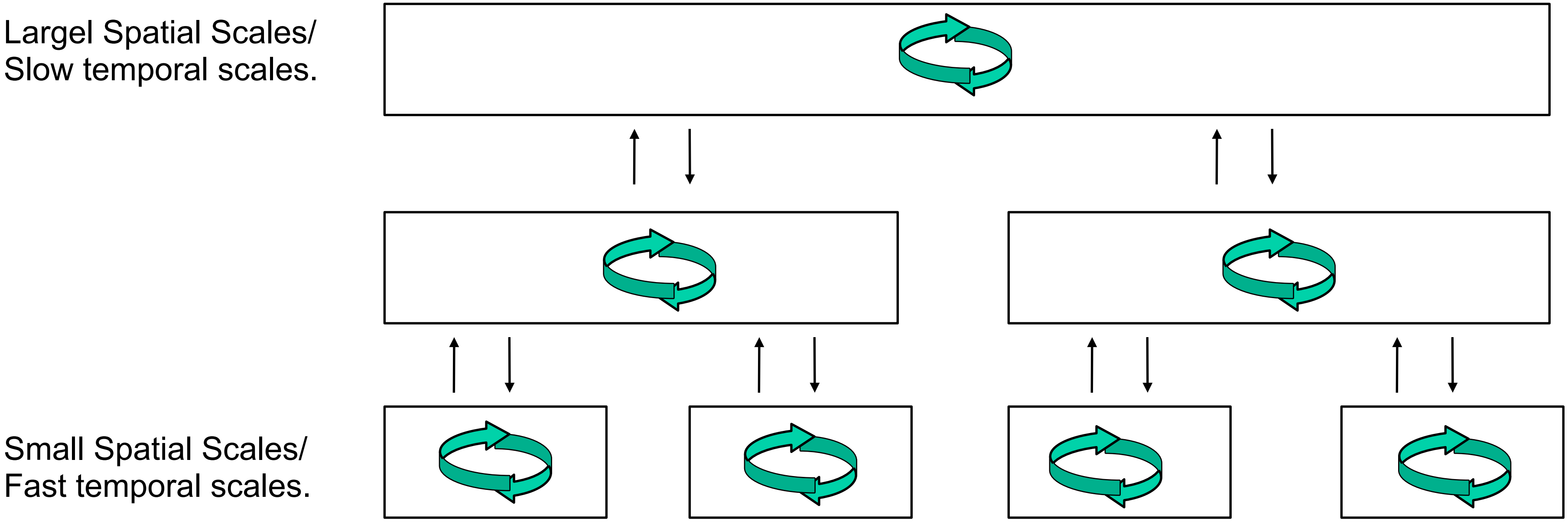
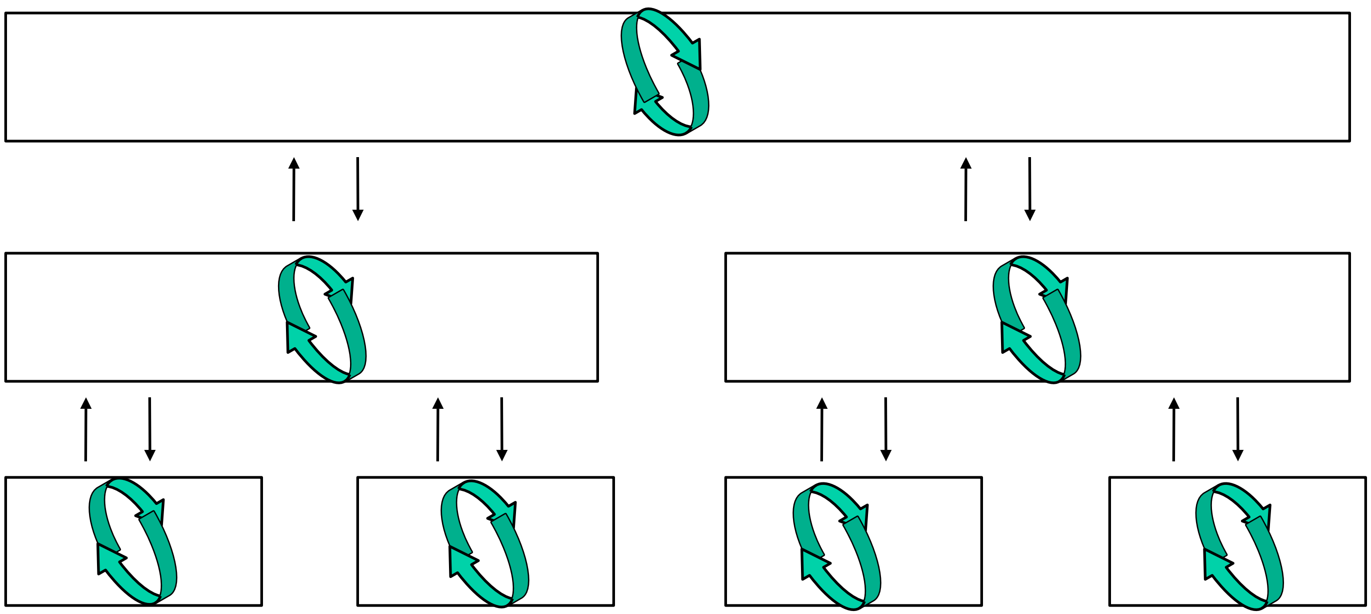


Spatial/temporal hierarchies in the world



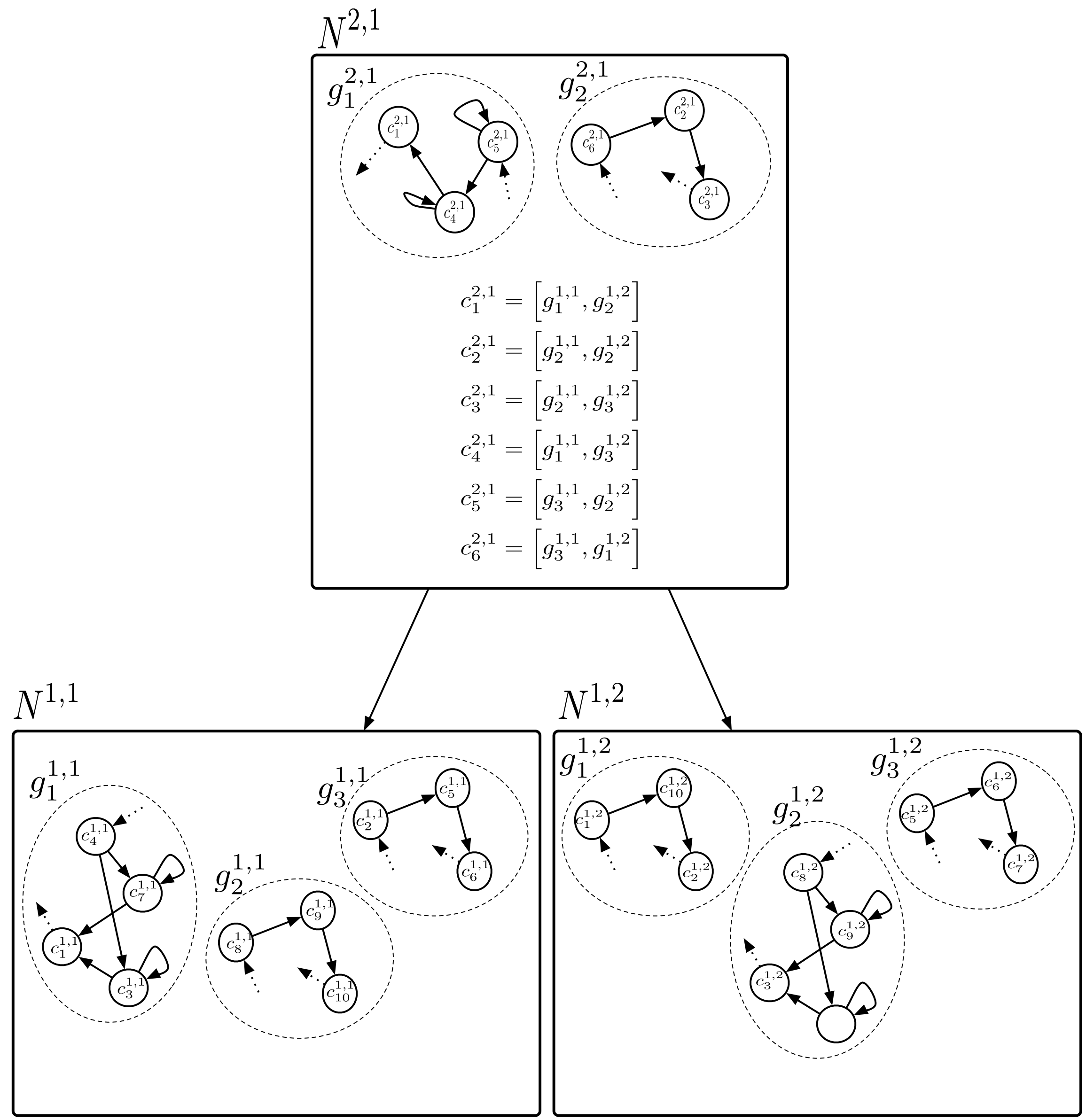
Spatial/temporal hierarchies in the world

Largel Spatial Scales/
Slow temporal scales.



Small Spatial Scales/
Fast temporal scales.

Generative Model : Hierarchical Temporal World



Learning in a node

- Level by level learning
- Each node
  - Stores co-occurrence patterns
  - Learns sequences

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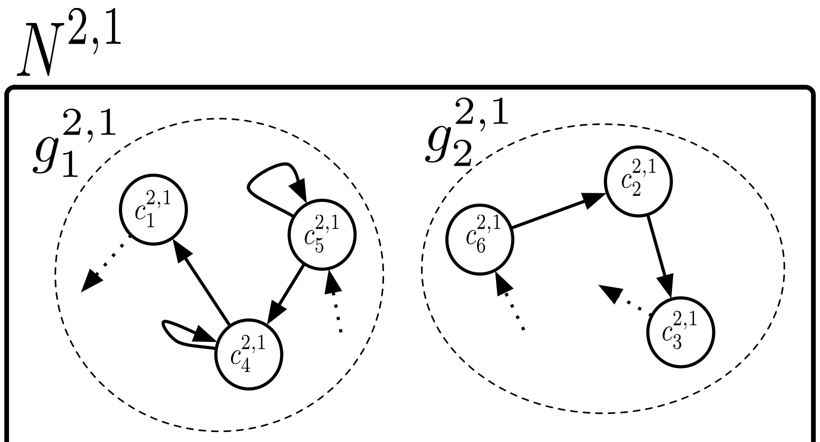

$$c_1^{2,1} = \left[g_1^{1,1}, g_2^{1,2}\right]$$
$$c_2^{2,1} = \left[g_2^{1,1}, g_2^{1,2}\right]$$
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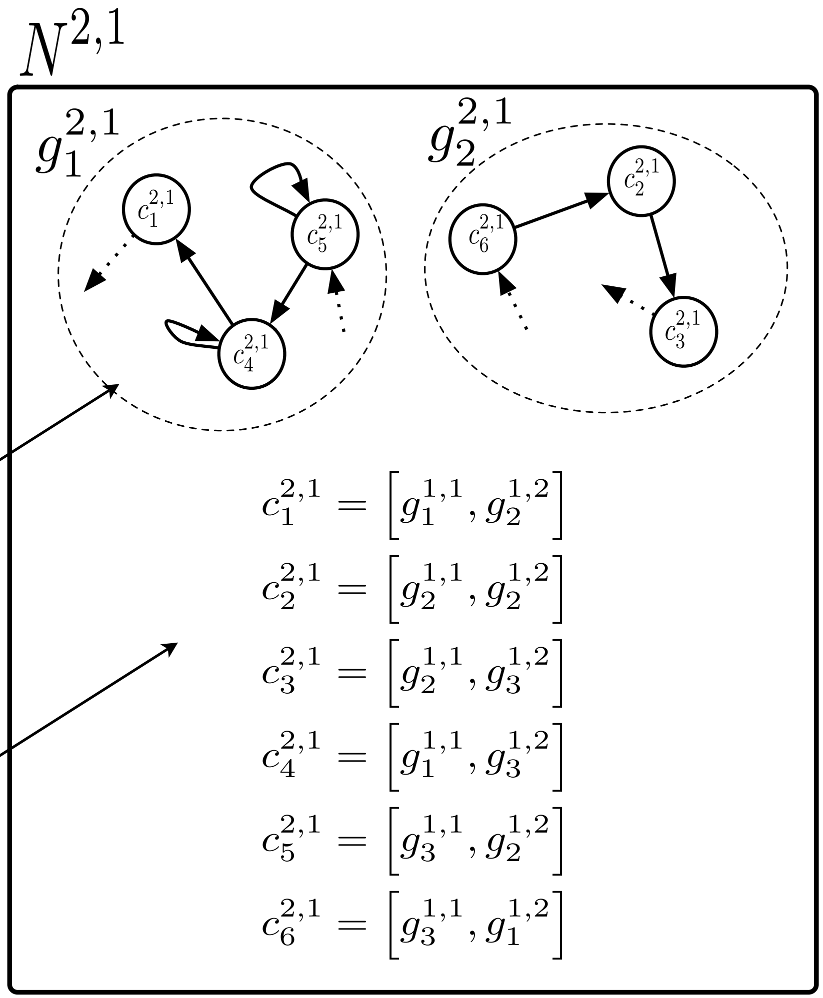


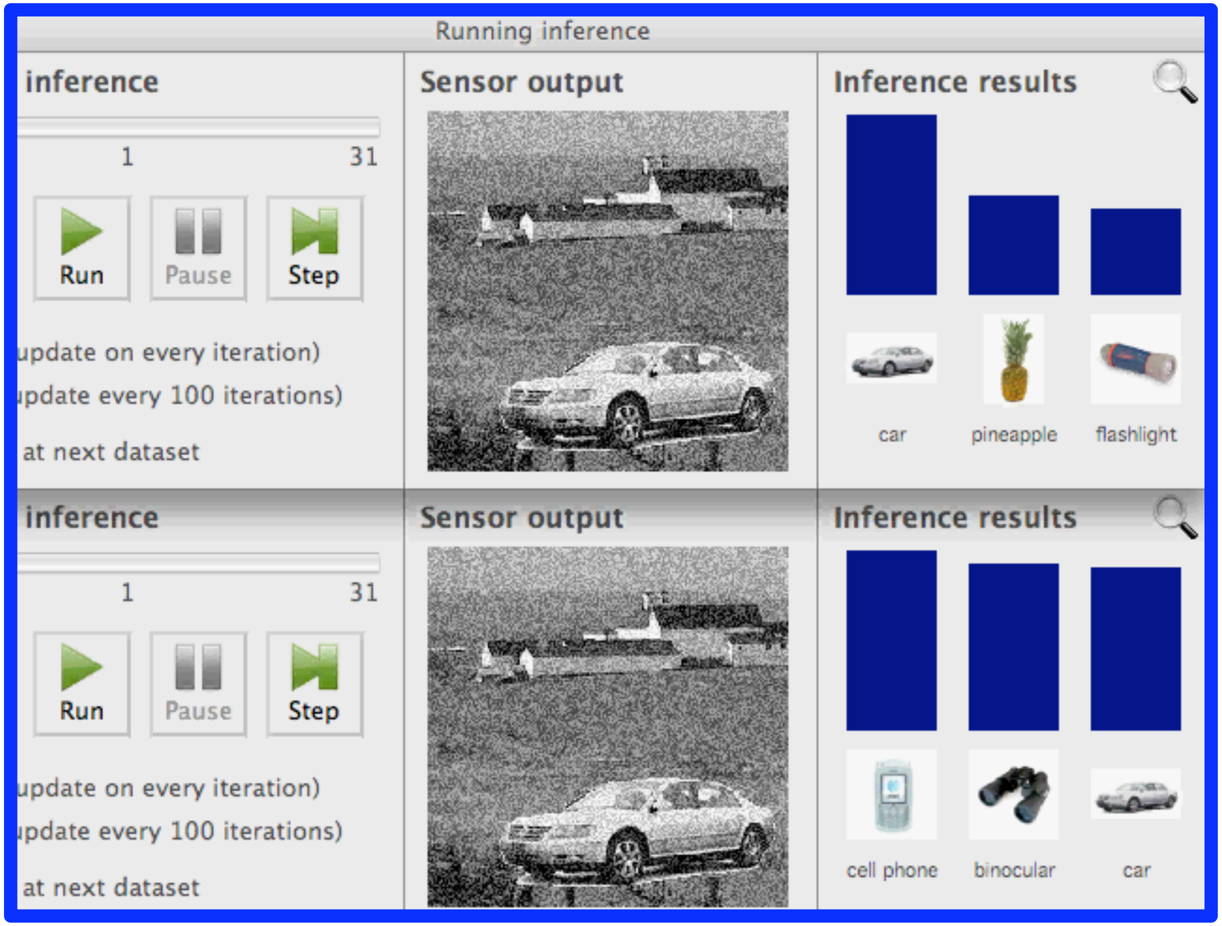
Markov chains (sequences)

Coincidence patterns

Demos

Video surveillance

Temporal Inference Demo (Preliminary Results)



← TBI

← non-TBI

What is happening behind the scenes?

Recognition/Prediction/Attention in HTMs

- Inference is done using Bayesian Belief Propagation on HTM hierarchy
  - *Probabilistic Reasoning in Intelligent Systems* by Judeal Pearl

Bayesian computation in the brain

Bayesian computation in the brain

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    - Lee and Mumford 2003

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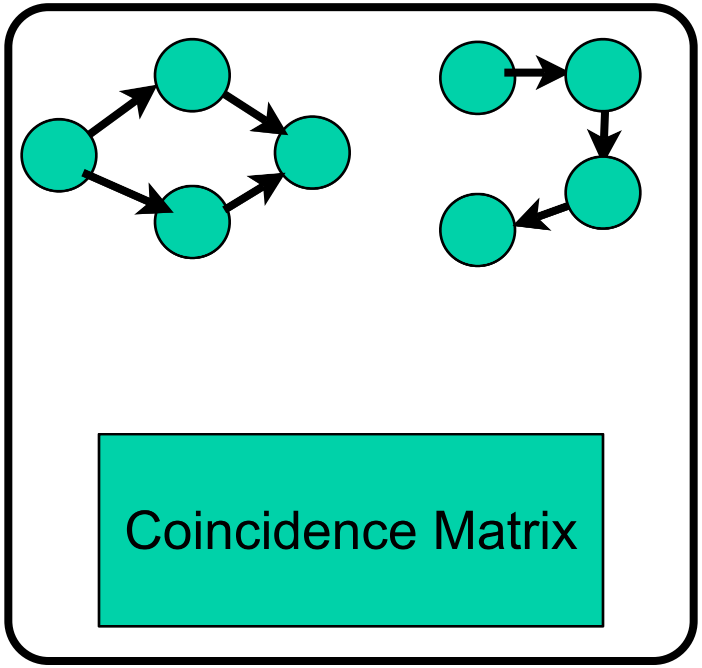
Bayesian computation in the brain

- Hierarchical Bayesian inference is gaining acceptance as the framework for understanding cortical computation
  - Lee and Mumford 2003
- "Bayesian framework is not yet a neural model. [Bayesian] framework currently helps explain the computations that underlie various brain functions, but not how the brain implements those computations"
  - Hegde & Felleman 2007

Our hypothesis

Bayesian Belief Propagation equations on HTMs correspond to the operations done by cortical microcircuits during inference.

Inference in an HTM node



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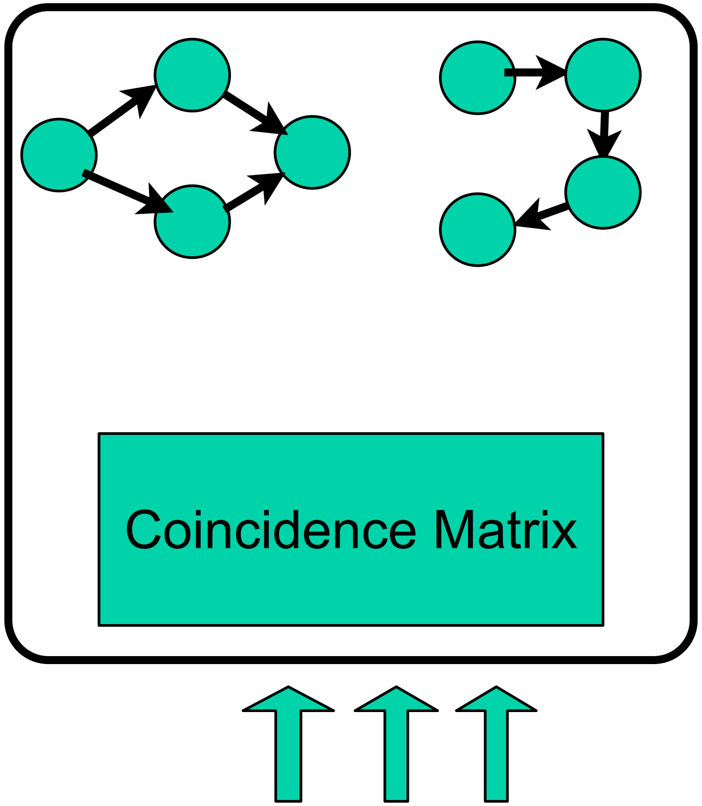

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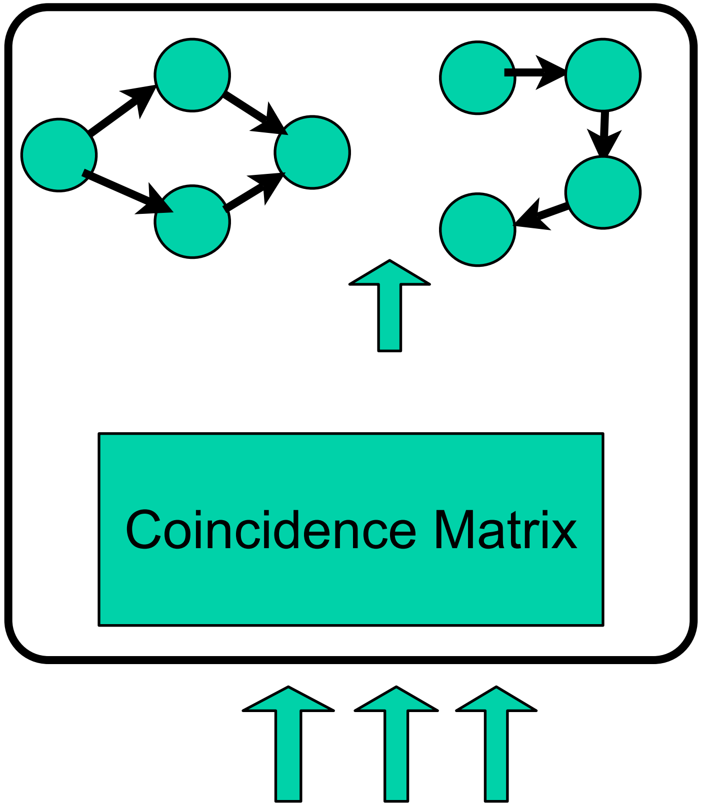

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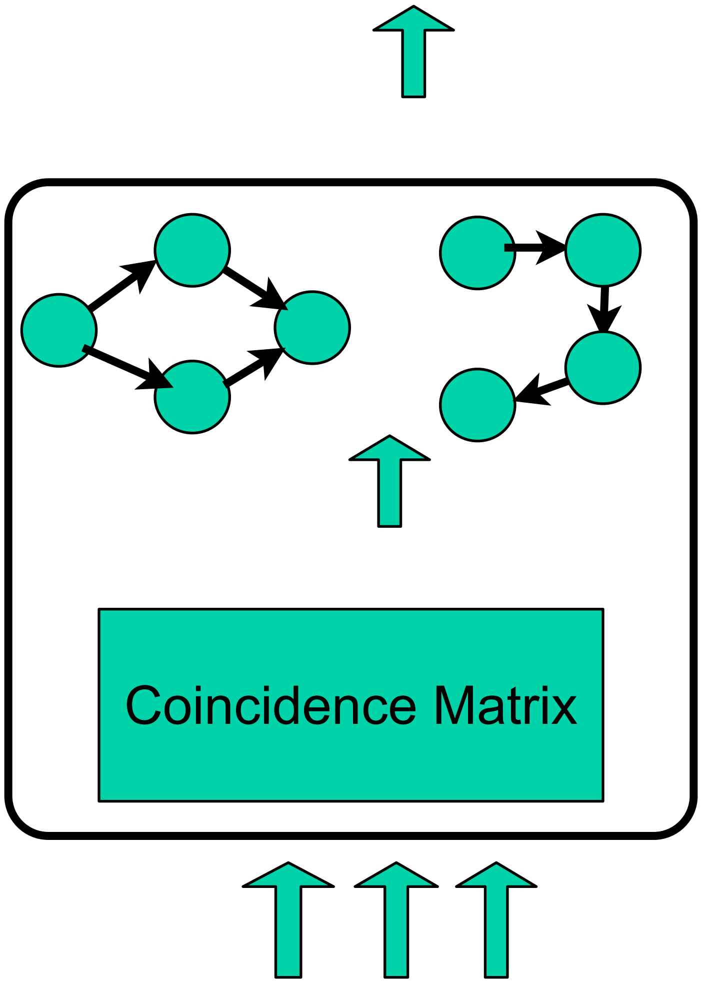

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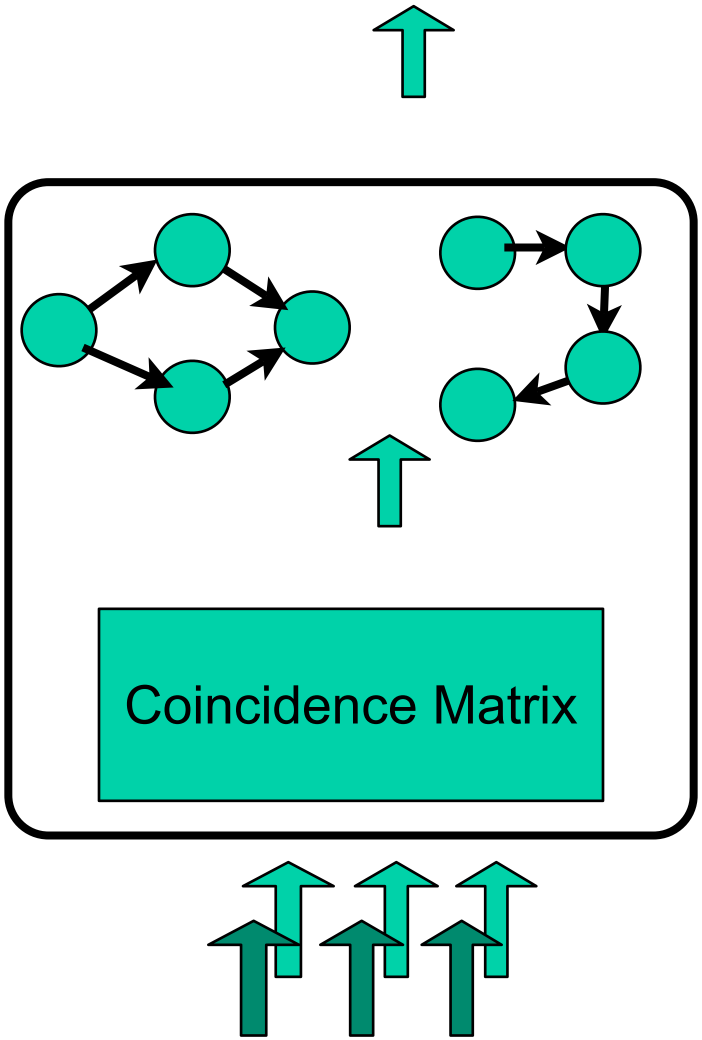

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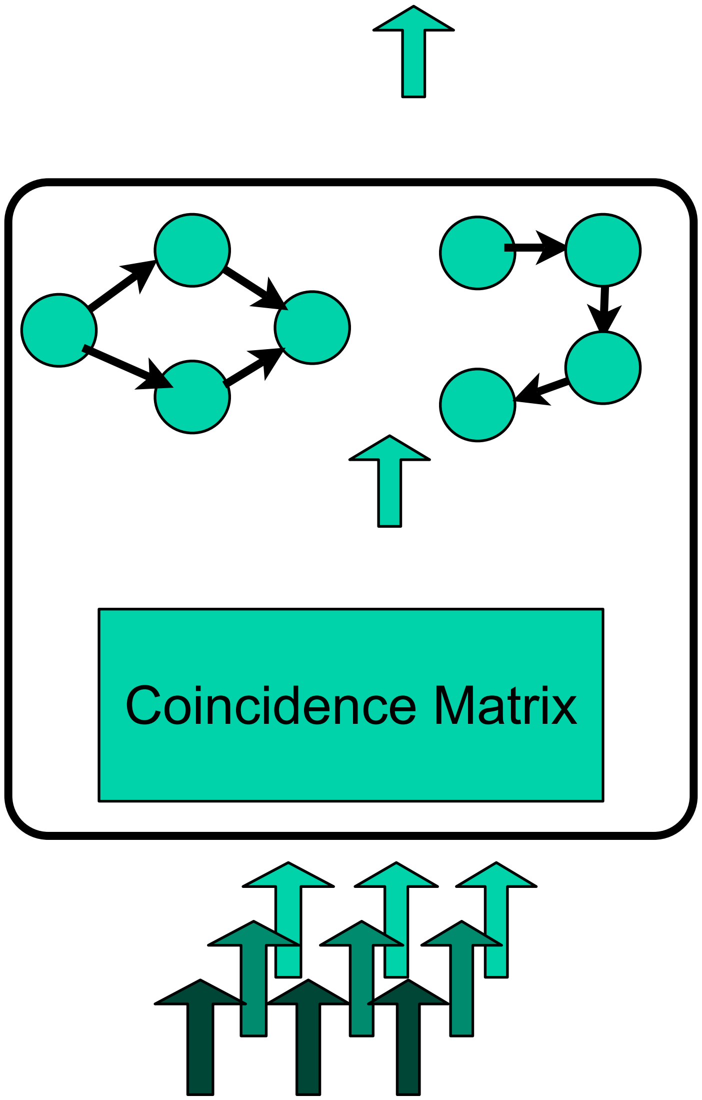

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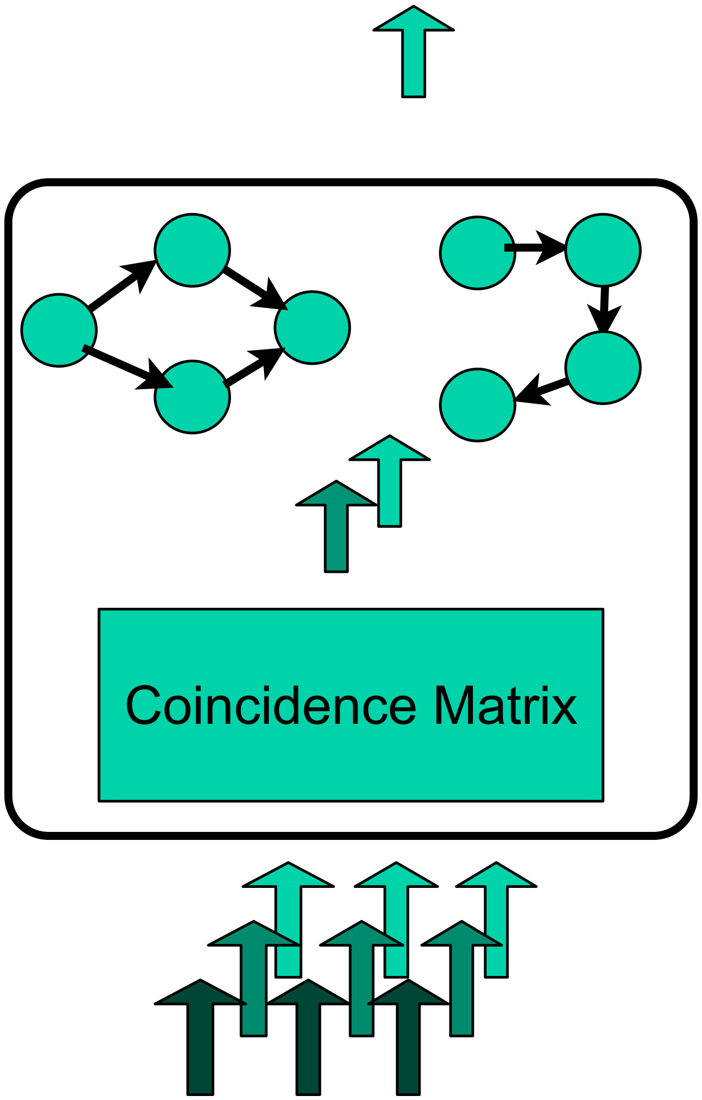

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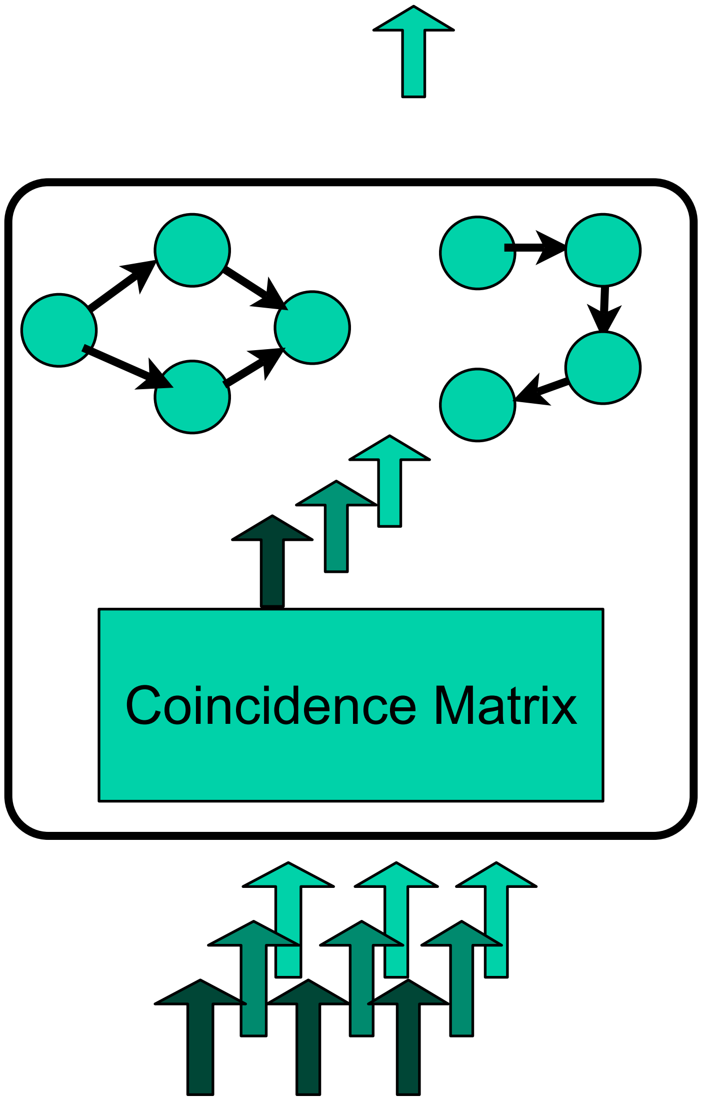

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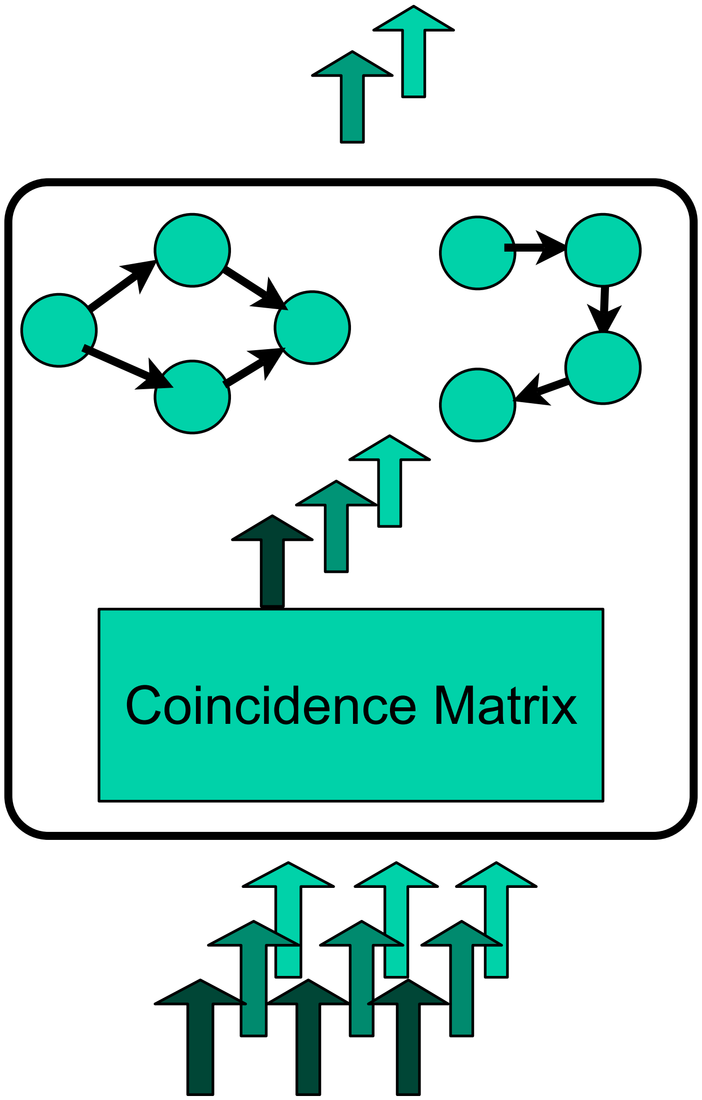

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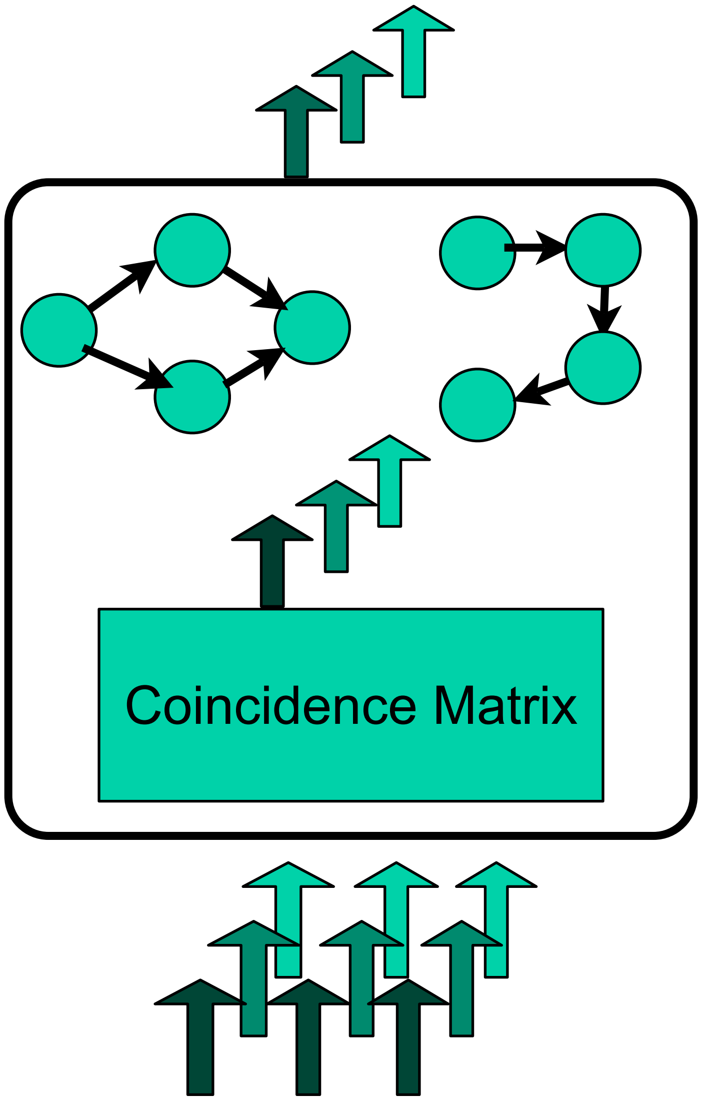

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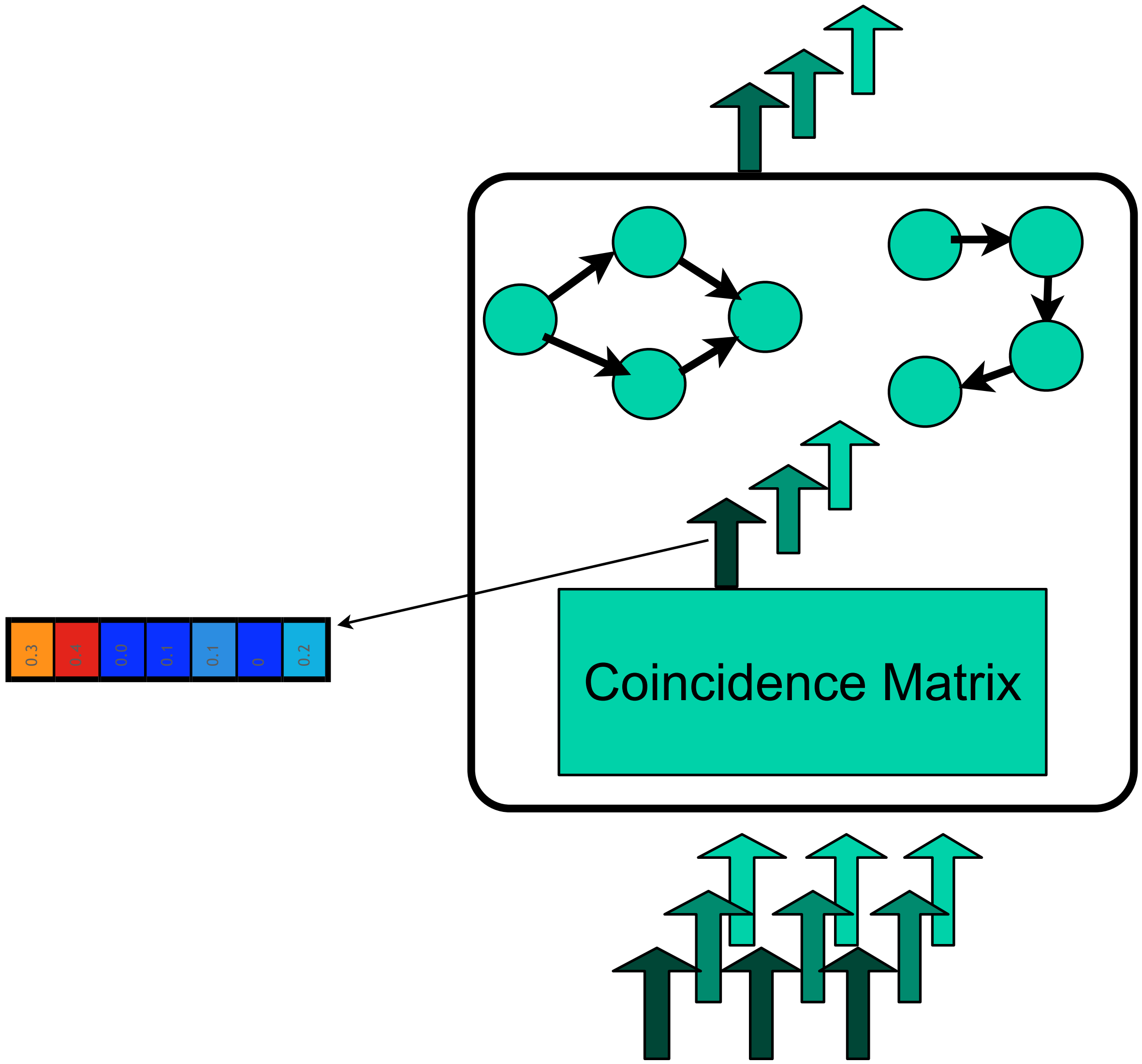

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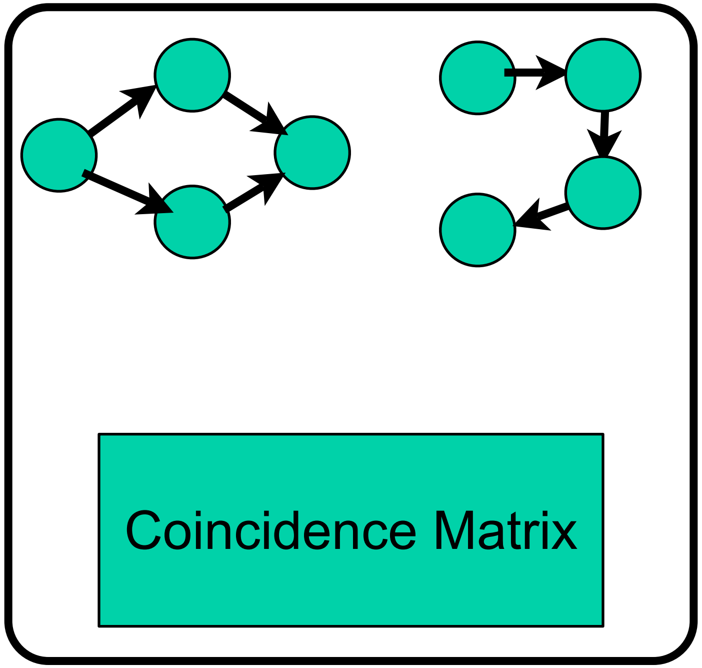

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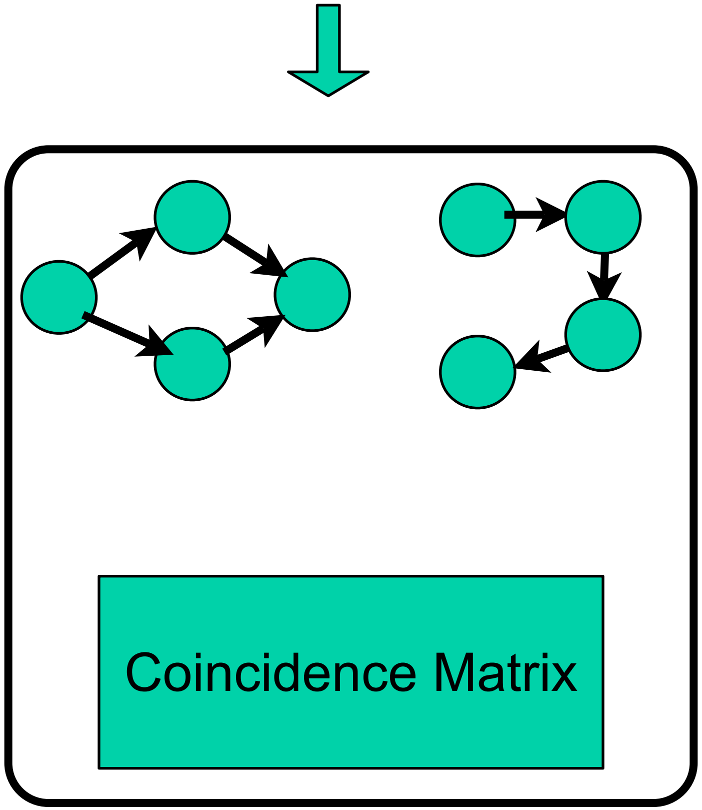

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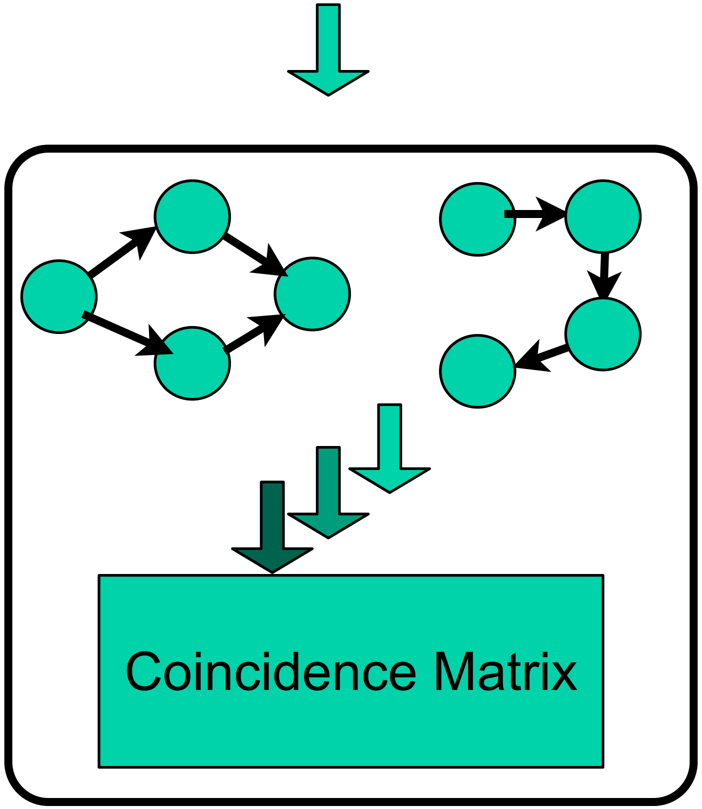

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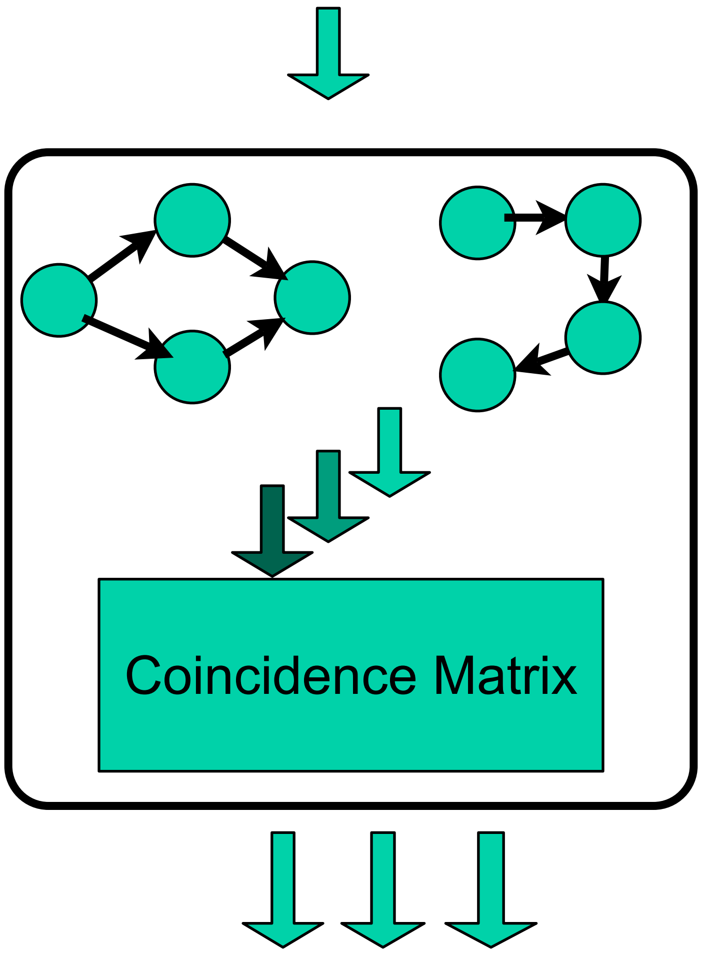

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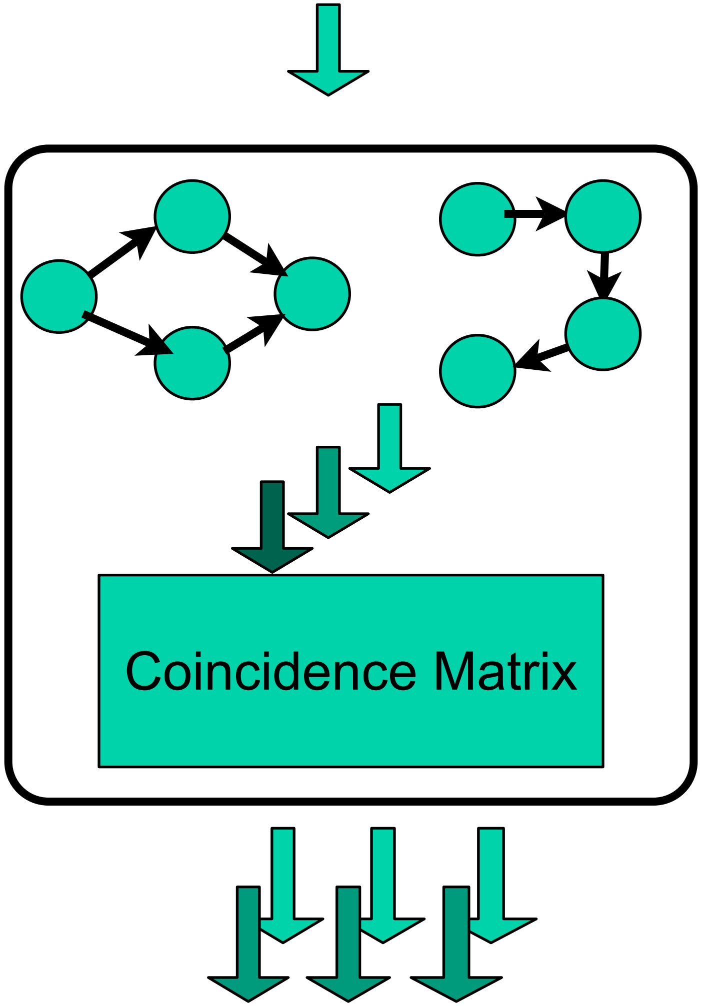

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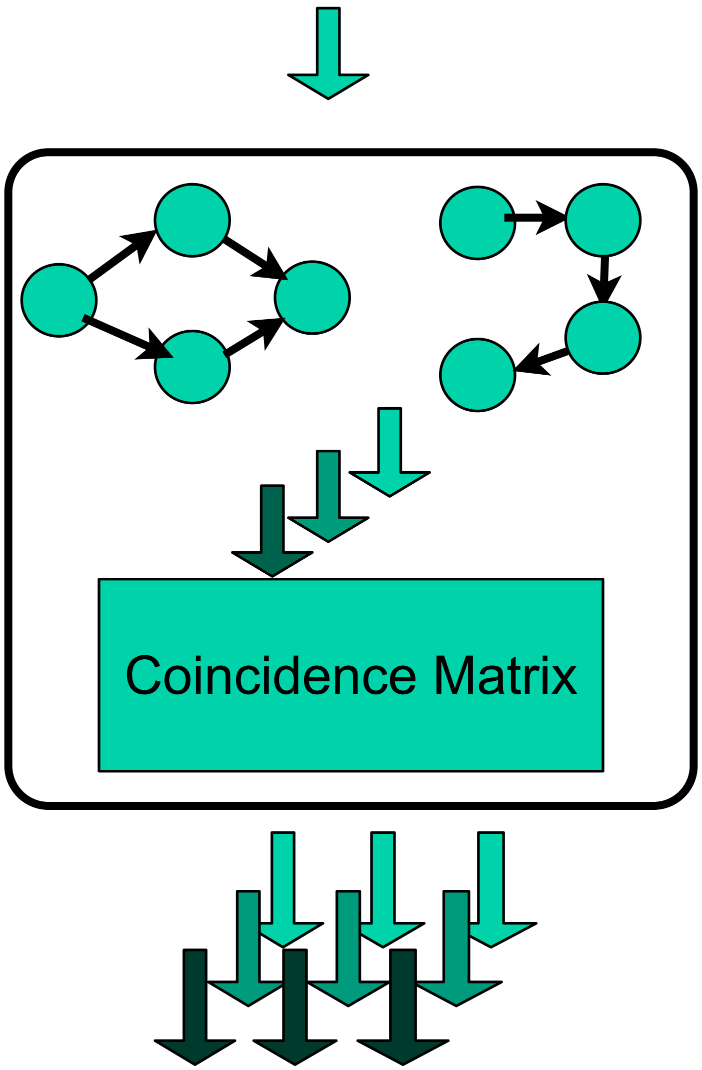

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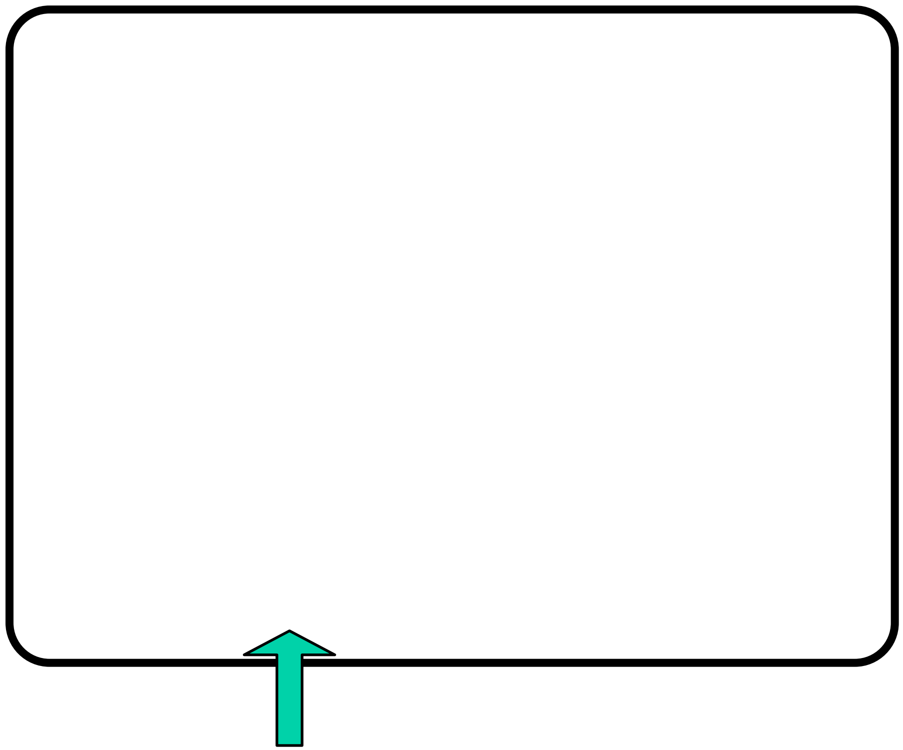

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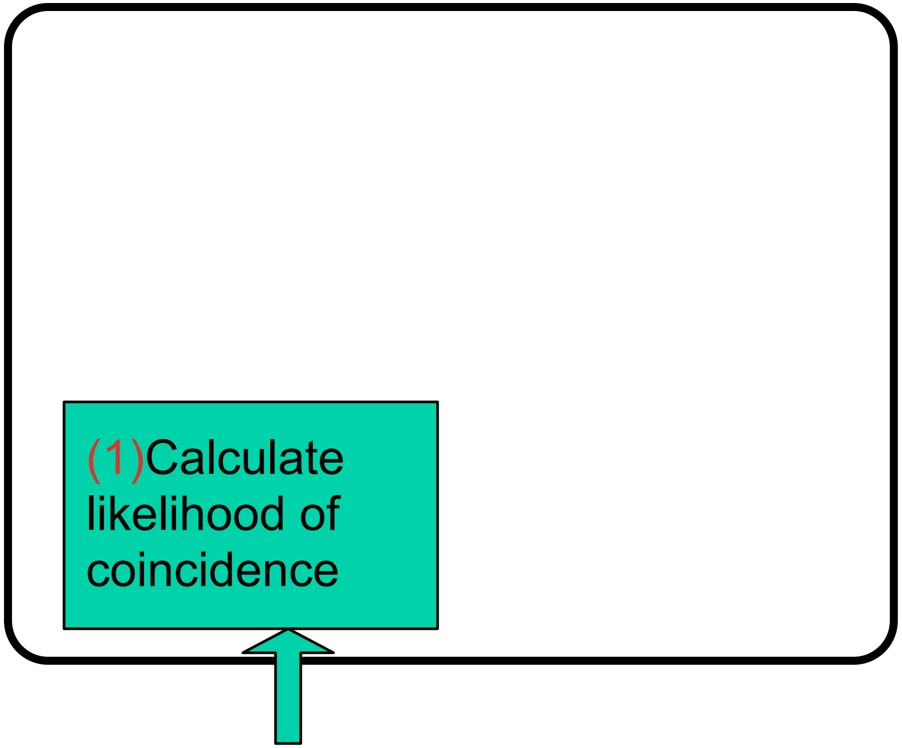

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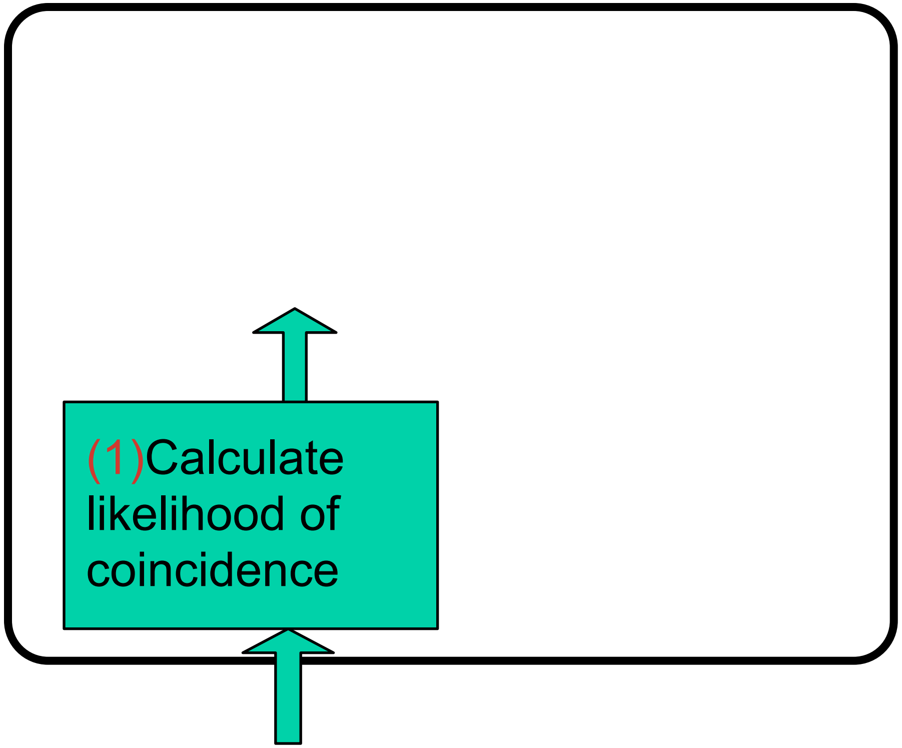

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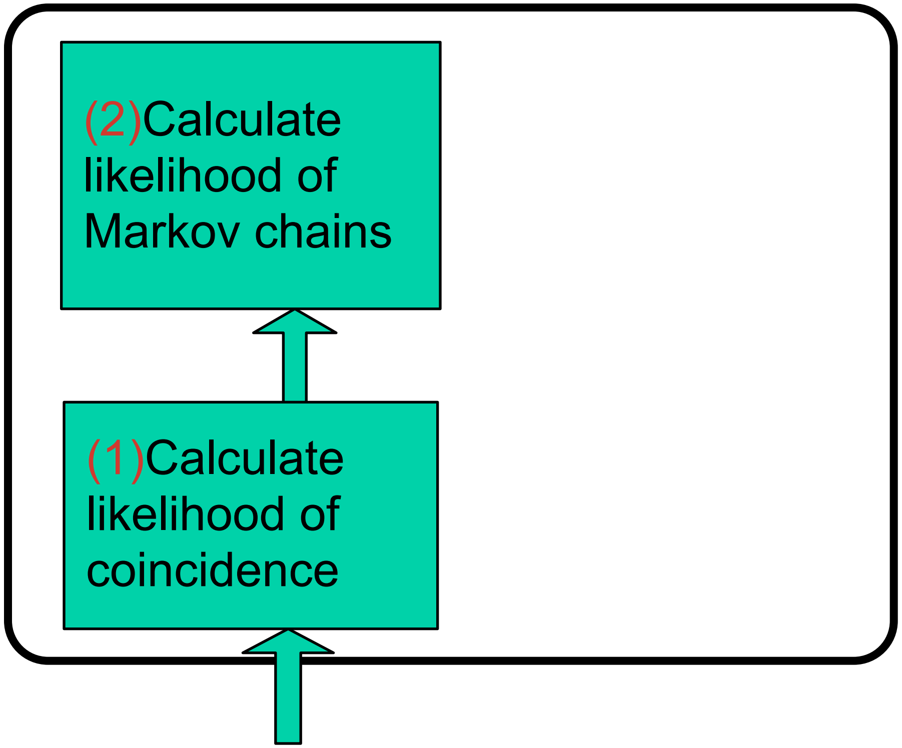

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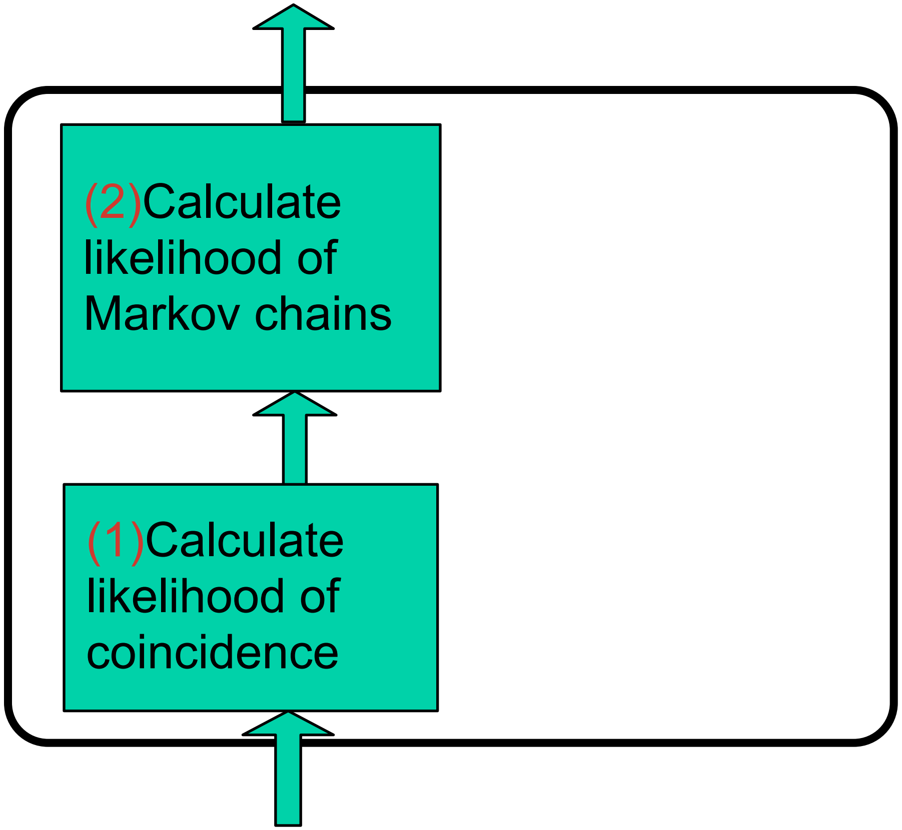

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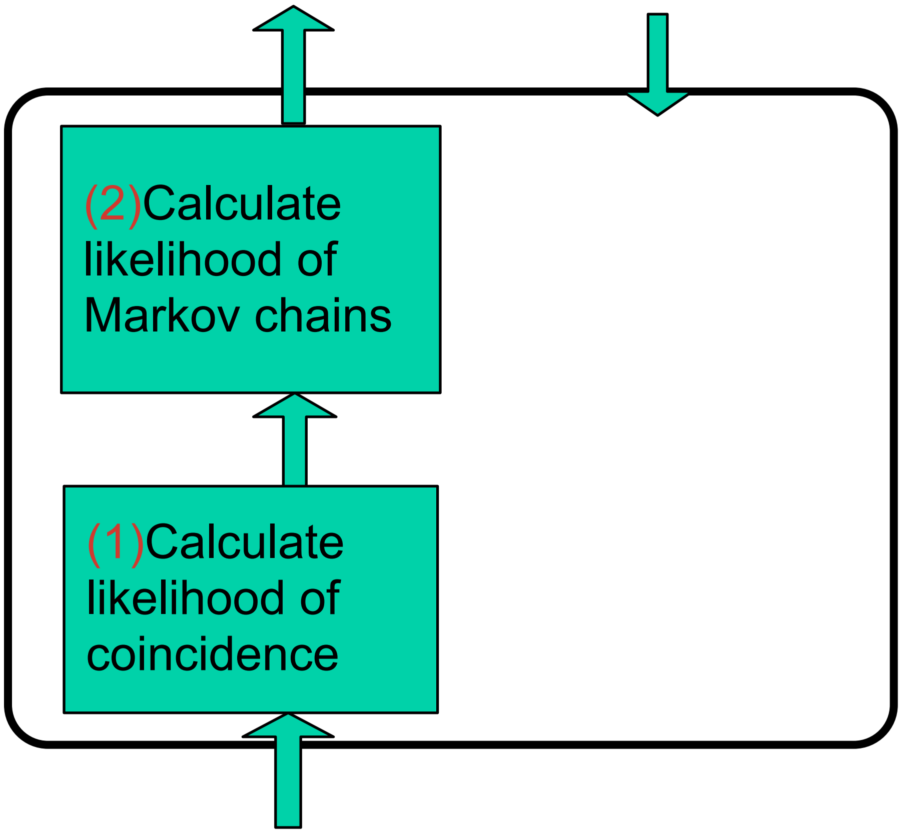

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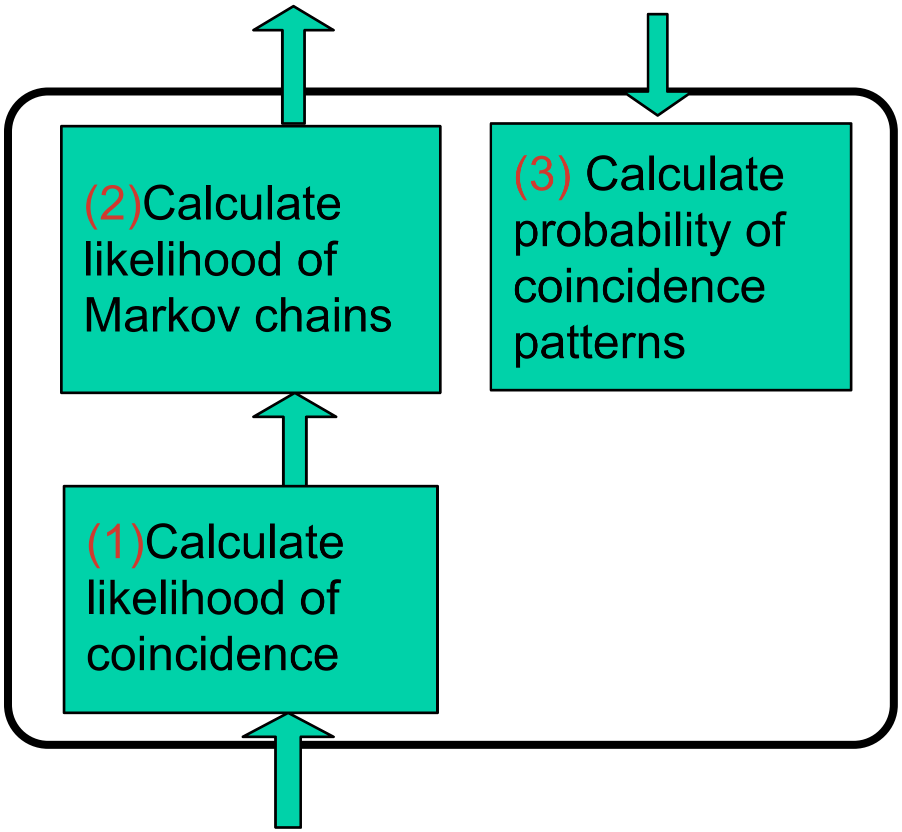

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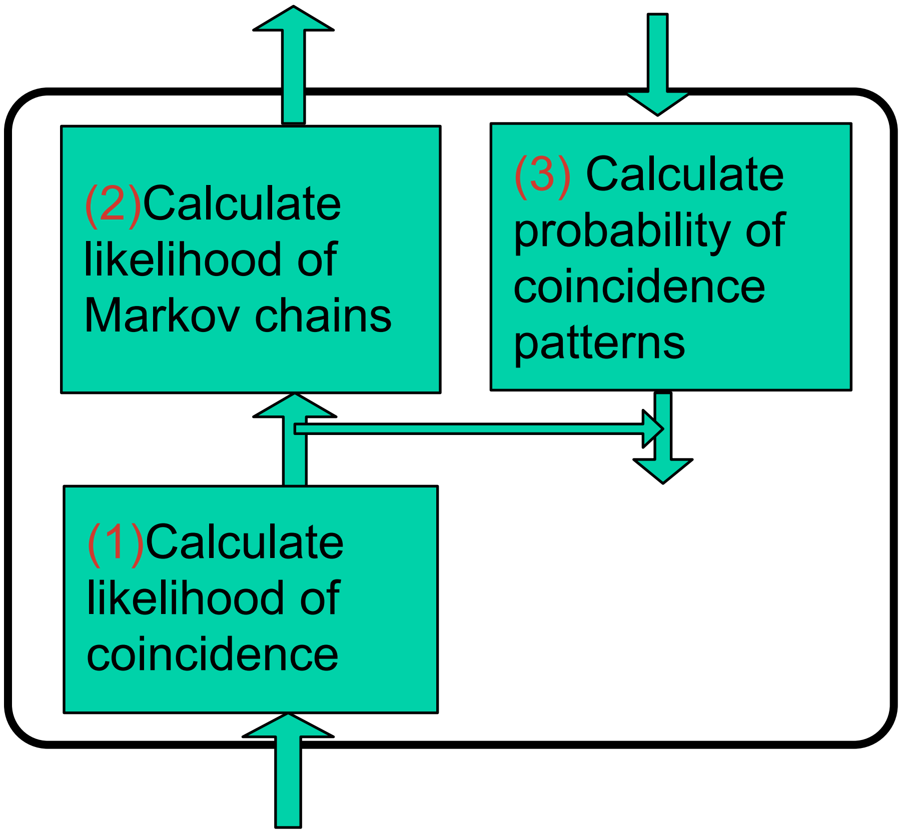

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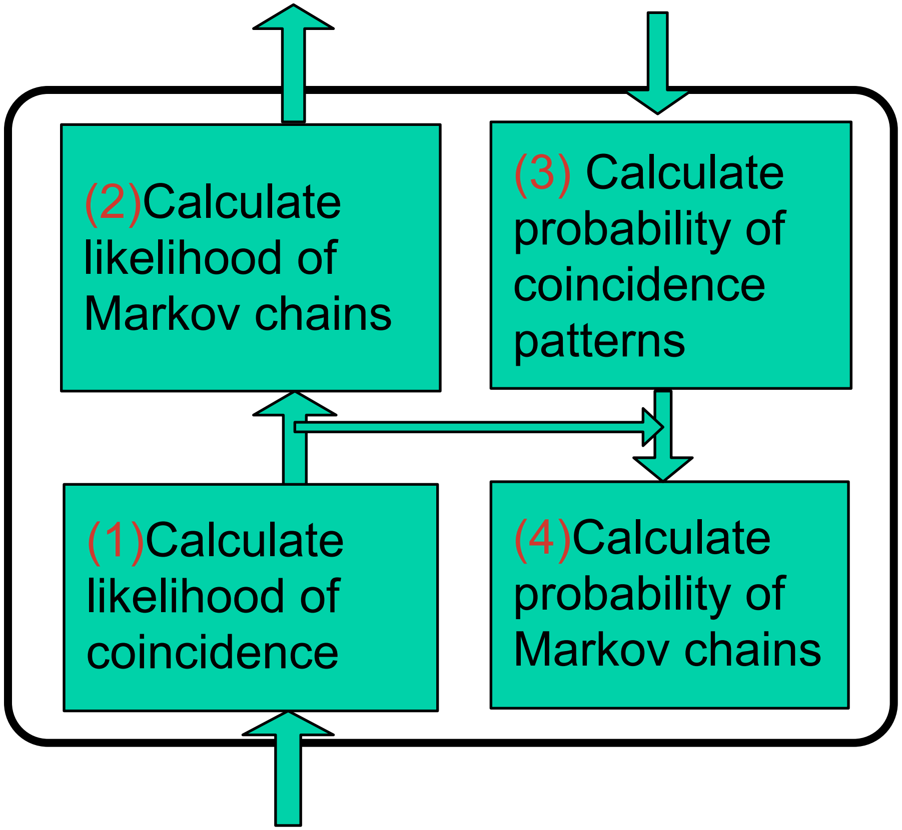

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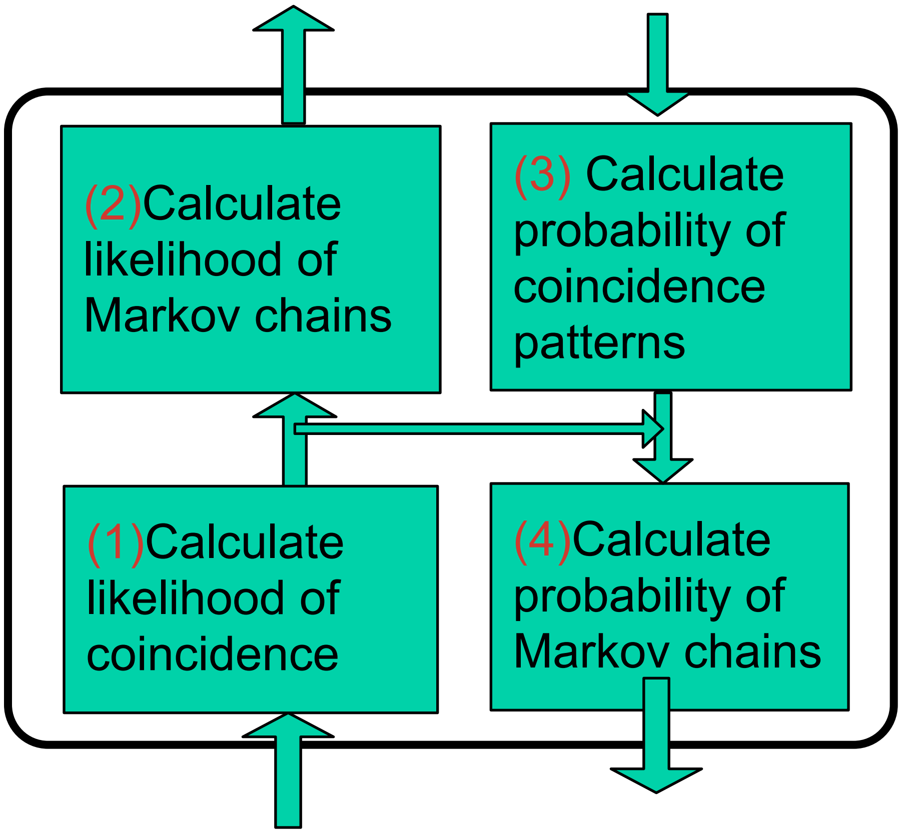

Belief propagation in HTMs : Equations

(1) Coincidence likelihood	$$y_t(i) = P(^-e_t \mid c_i(t)) \propto \prod_{j=1}^{M} \lambda_t^{m_j}(r_i^{m_j}) \quad (1)$$ where coincidence-pattern c_i is the co-occurrence of $r_i^{m_1}$'th Markov chain from child 1, $r_i^{m_2}$'th Markov chain from child 2, $\cdots$, and $r_i^{m_M}$'th Markov chain from child M.
(2) Markov chain likelihood	$$\lambda_t^k(g_r) = P(^-e_0^t \mid g_r(t)) \propto \sum_{c_i(t)\in C^k} \alpha_t(c_i, g_r) \quad (2)$$ $$\alpha_t(c_i, g_r) = P(^-e_t \mid c_i(t)) \sum_{c_j(t-1)\in C^k} P(c_i(t) \mid c_j(t-1), g_r)\alpha_{t-1}(c_j, g_r) \quad (3)$$ $$\alpha_0(c_i, g_r) = P(^-e_0 \mid c_i(t=0))P(c_i(t=0) \mid g_r) \quad (4)$$
(3) Coincidence Belief	$$Bel_t(c_i) \propto \sum_{g_r\in G^k} P(g_r \mid ^+e_0)\beta_t(c_i, g_r) \quad (5)$$ $$\beta_t(c_i, g_r) = P(^-e_t \mid c_i(t)) \sum_{c_j(t-1)\in C^k} P(c_i(t) \mid c_j(t-1), g_r)\beta_{t-1}(c_j, g_r) \quad (6)$$ $$\beta_0(c_i, g_r) = P(^-e_0 \mid c_i(t=0))P(c_i(t=0) \mid g_r, ^+e_0) \quad (7)$$
(4) Feedback messages	$$\pi^{child}(g_m) \propto \sum_{i\forall i} I(c_i)Bel(c_i) \quad (8)$$ where $$I(c_i) = \begin{cases} 1, & \text{if } g_m^{child} \text{ is a component of } c_i \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

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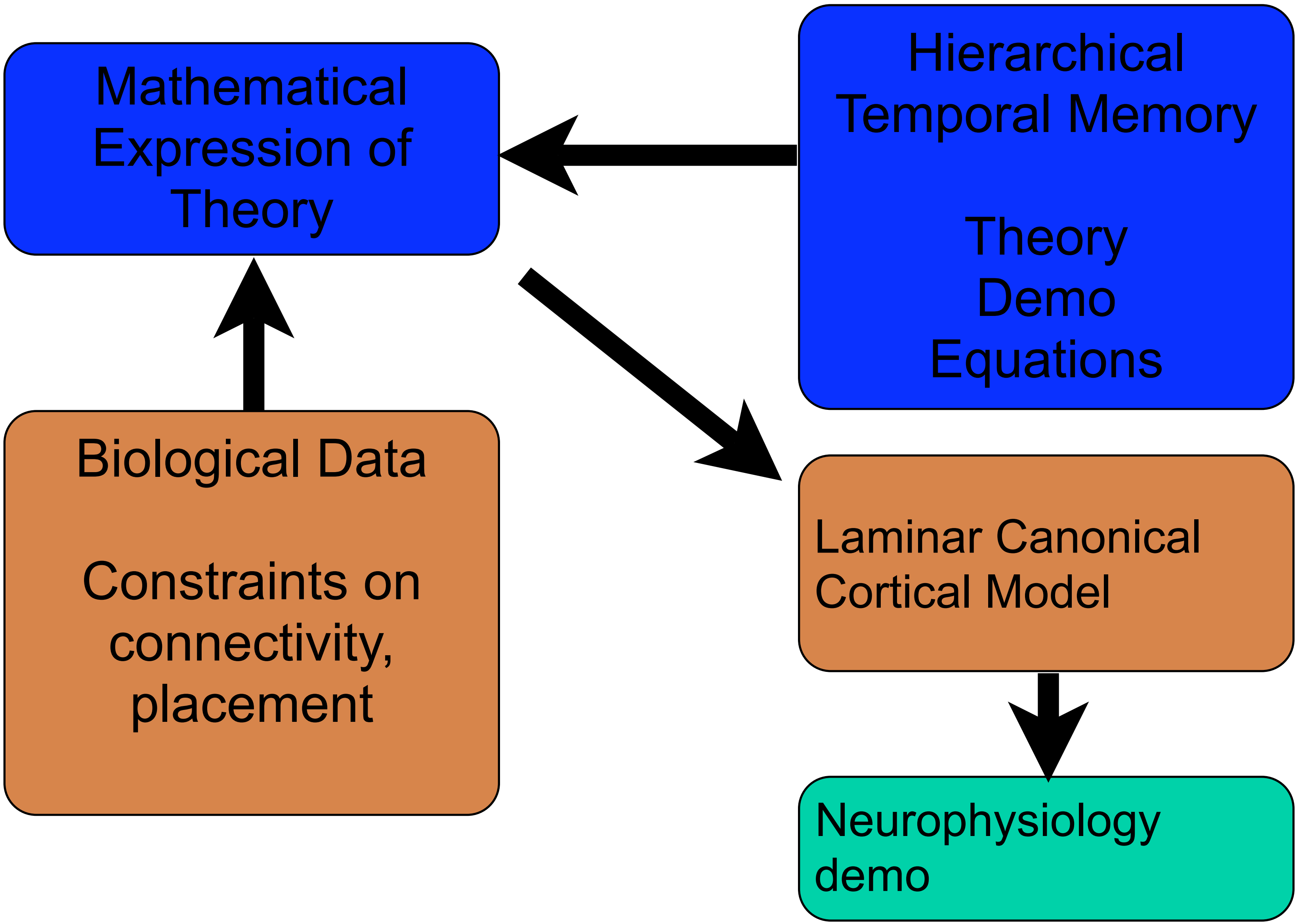
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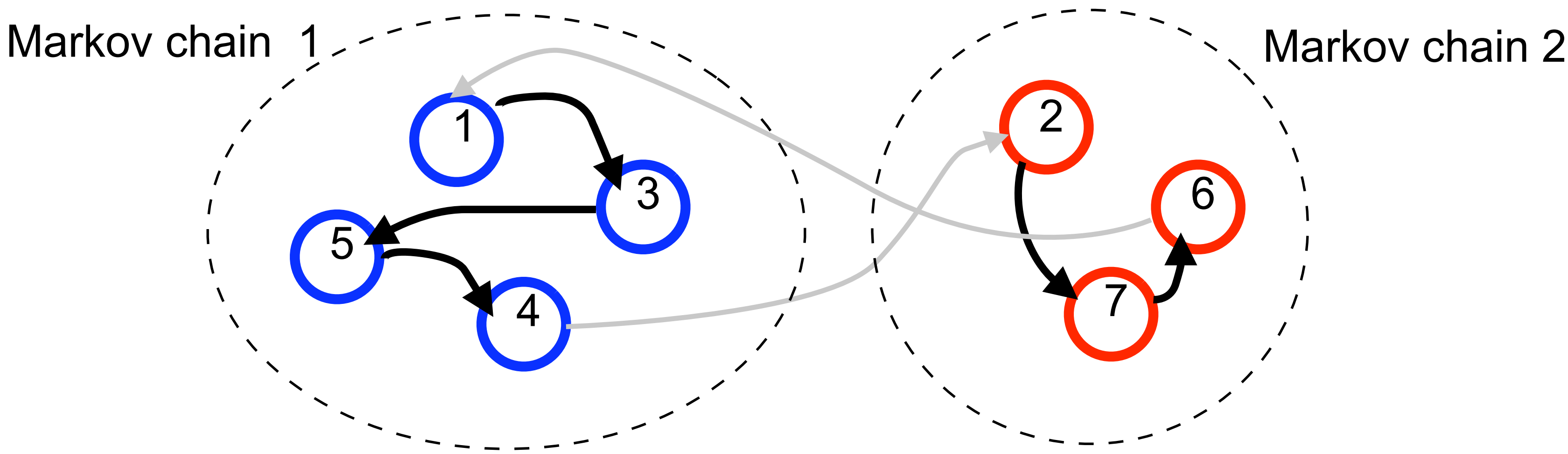
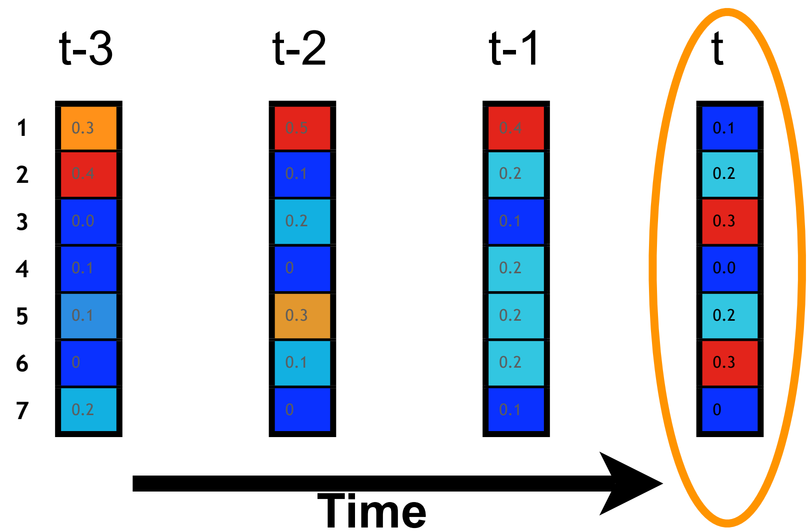
Proposal:

A set of layer 2/3 neurons implement sequence inference using dynamic programming

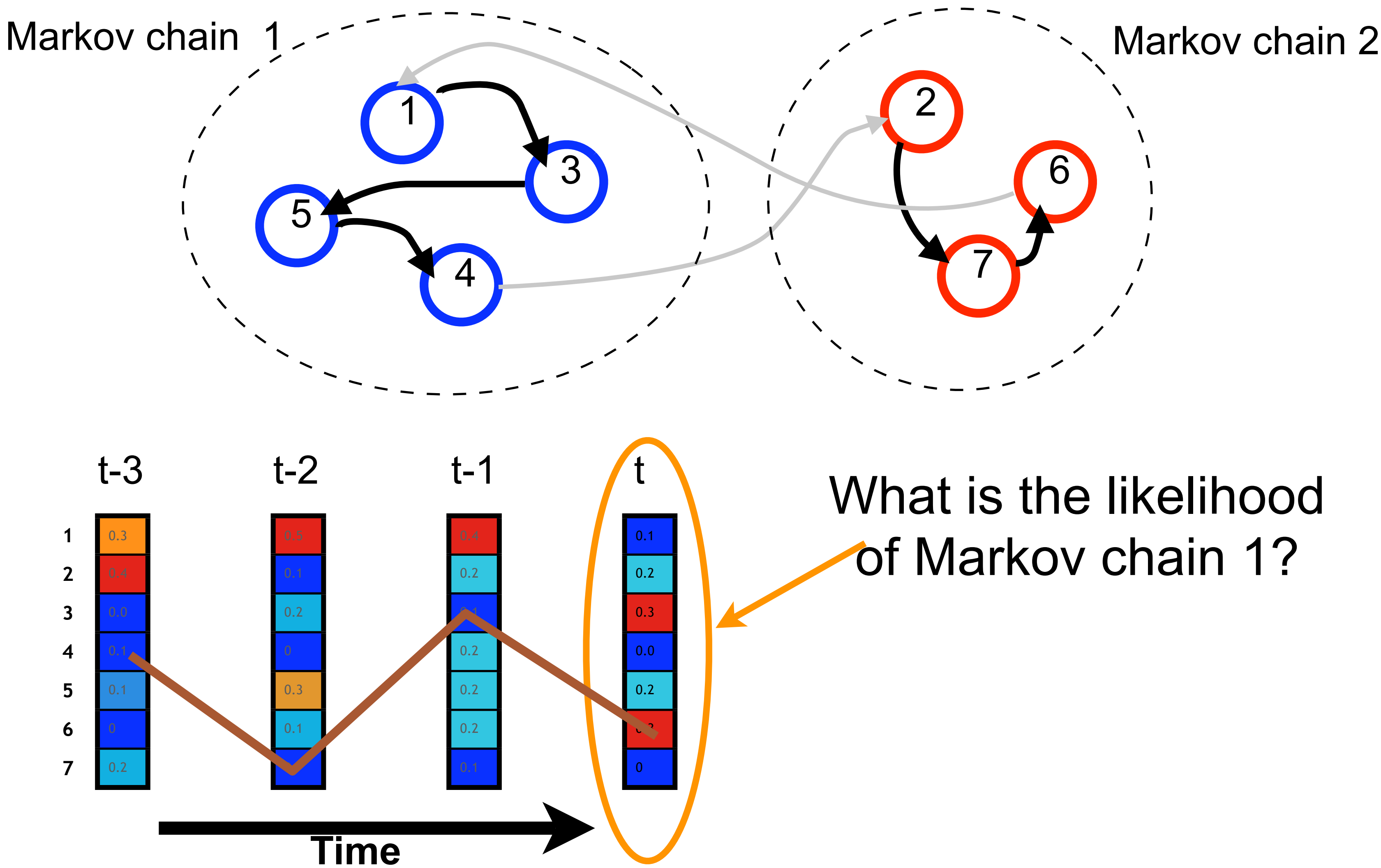
Dynamic programming for Markov chain inference



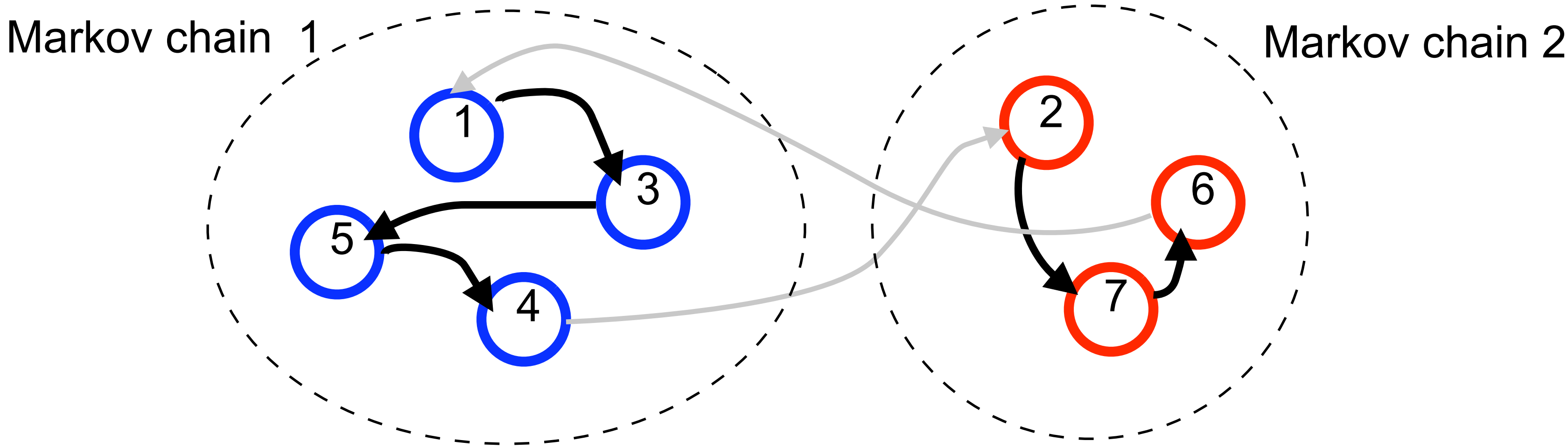


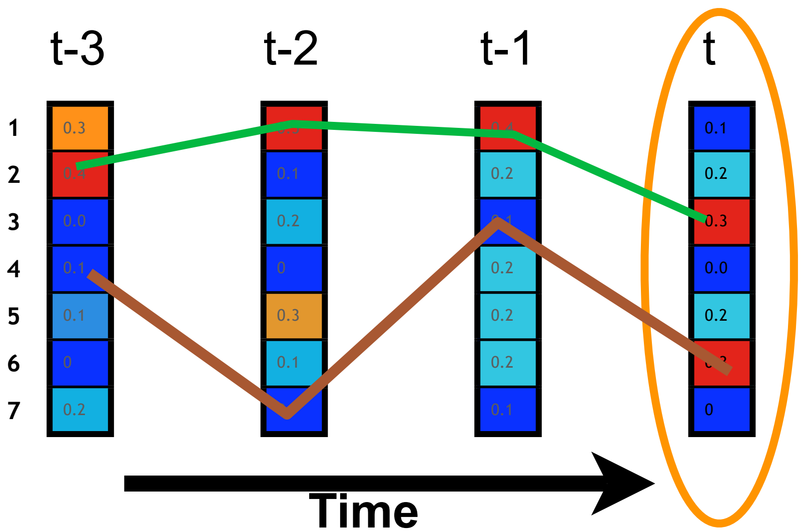
What is the likelihood of Markov chain 1?

Dynamic programming for Markov chain inference



Dynamic programming for Markov chain inference





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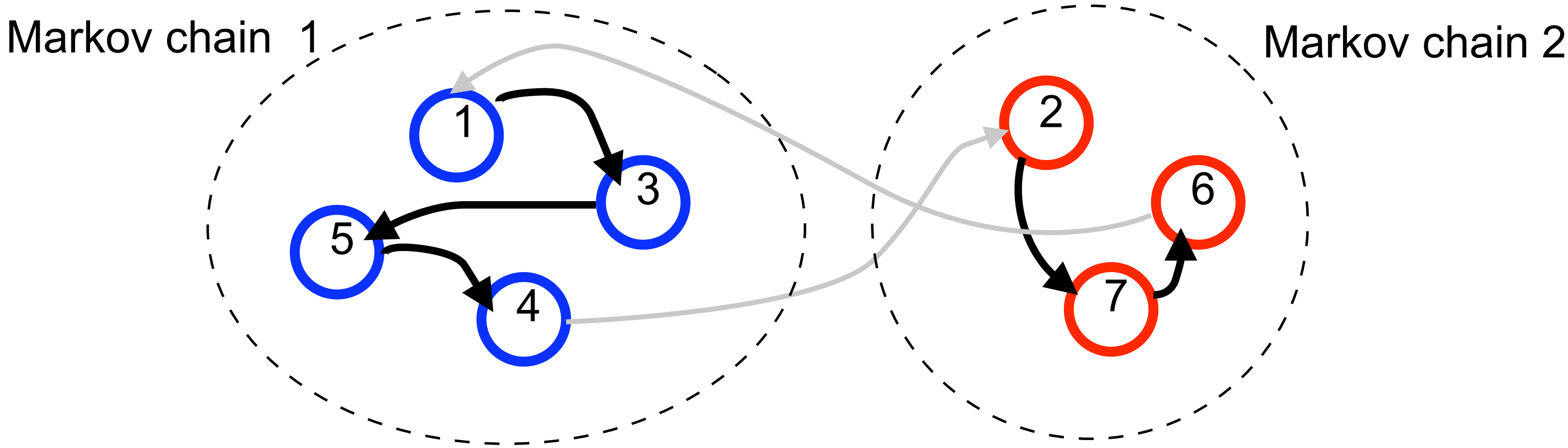

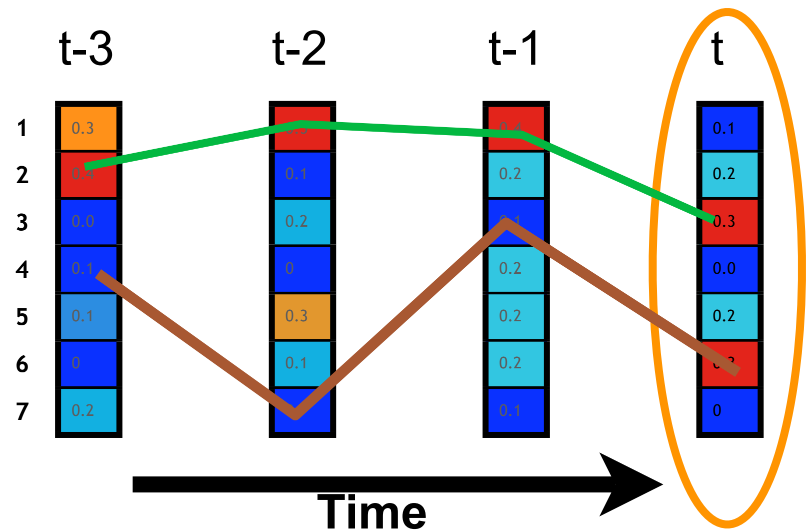

What is the likelihood of Markov chain 1?

Dynamic programming calculates these likelihoods efficiently

Calculate the likelihood of Markov chains

$$\alpha_t(c_i, g_r) = P(^{-}e_t | c_i(t)) \sum_{c_j(t-1) \in C^k} P(c_i(t) | c_j(t-1), g_r) \alpha_{t-1}(c_j, g_r)$$

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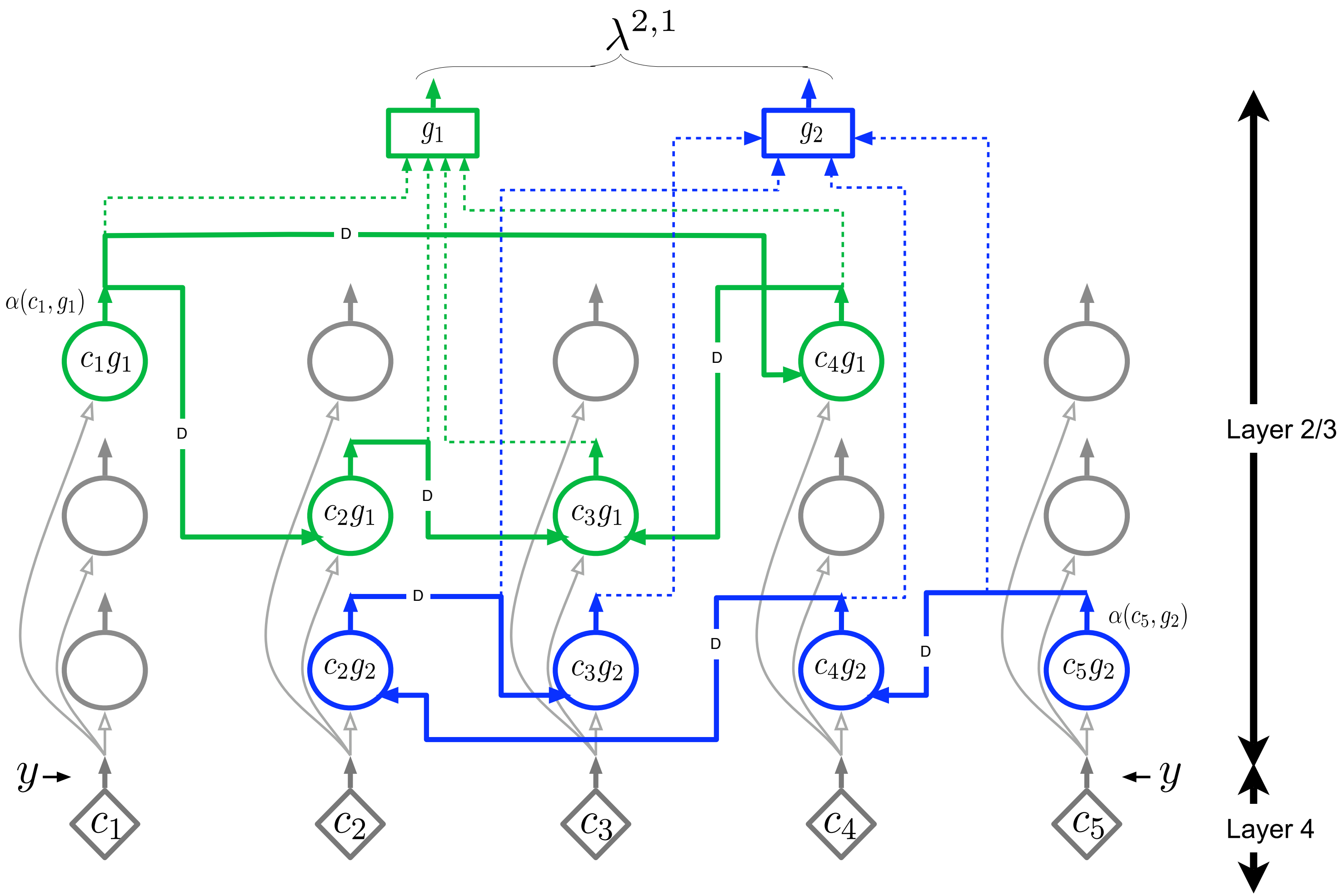

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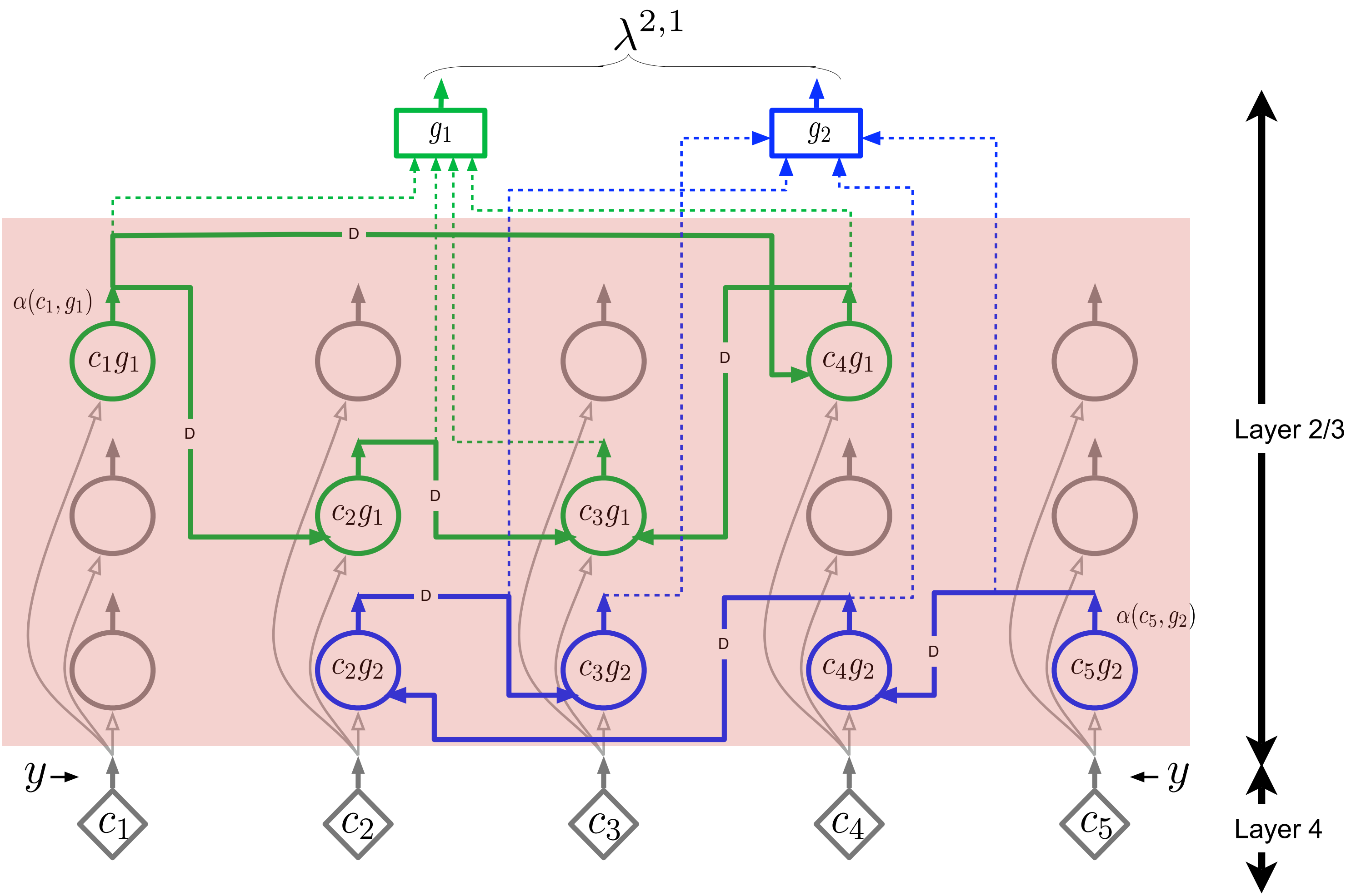

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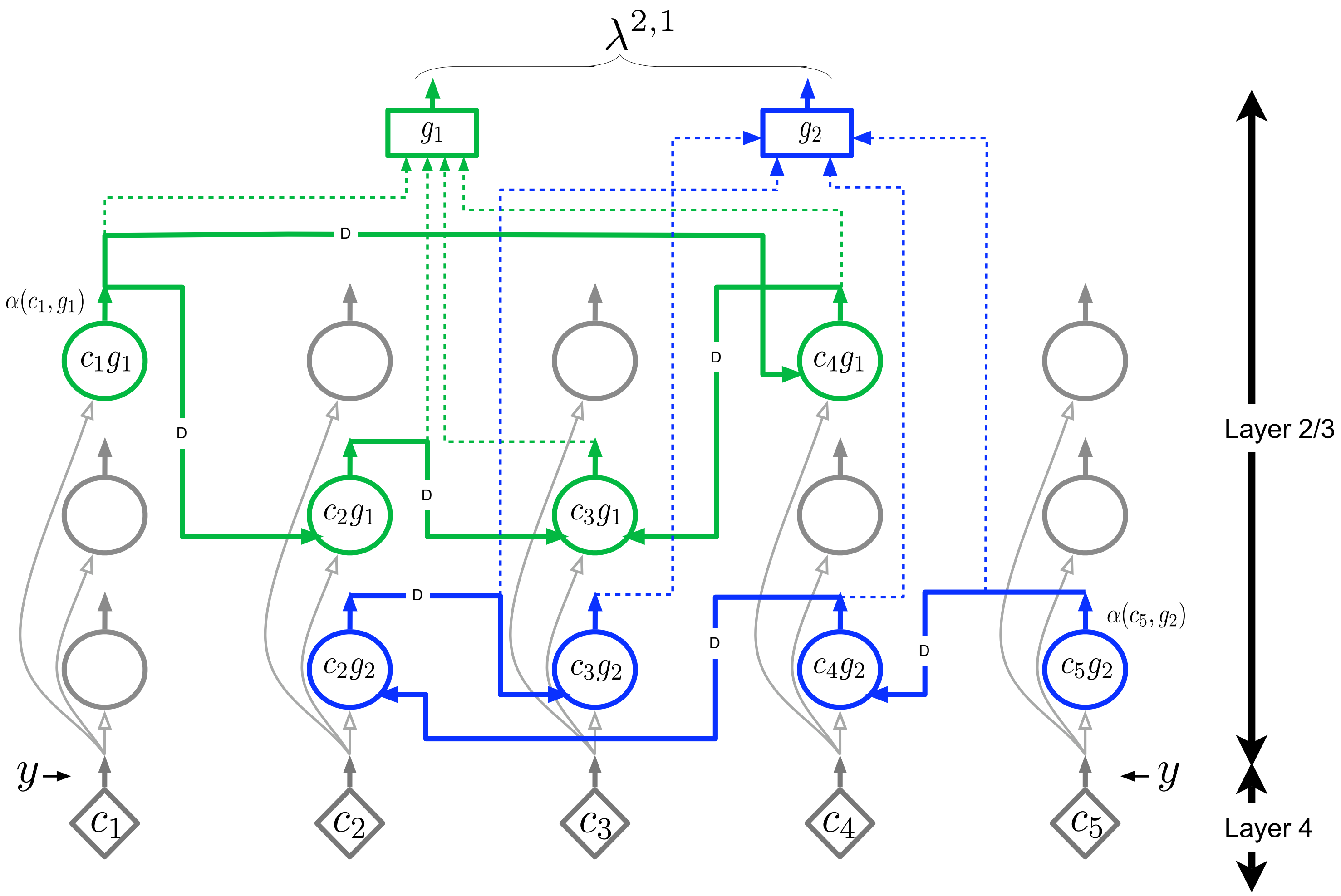

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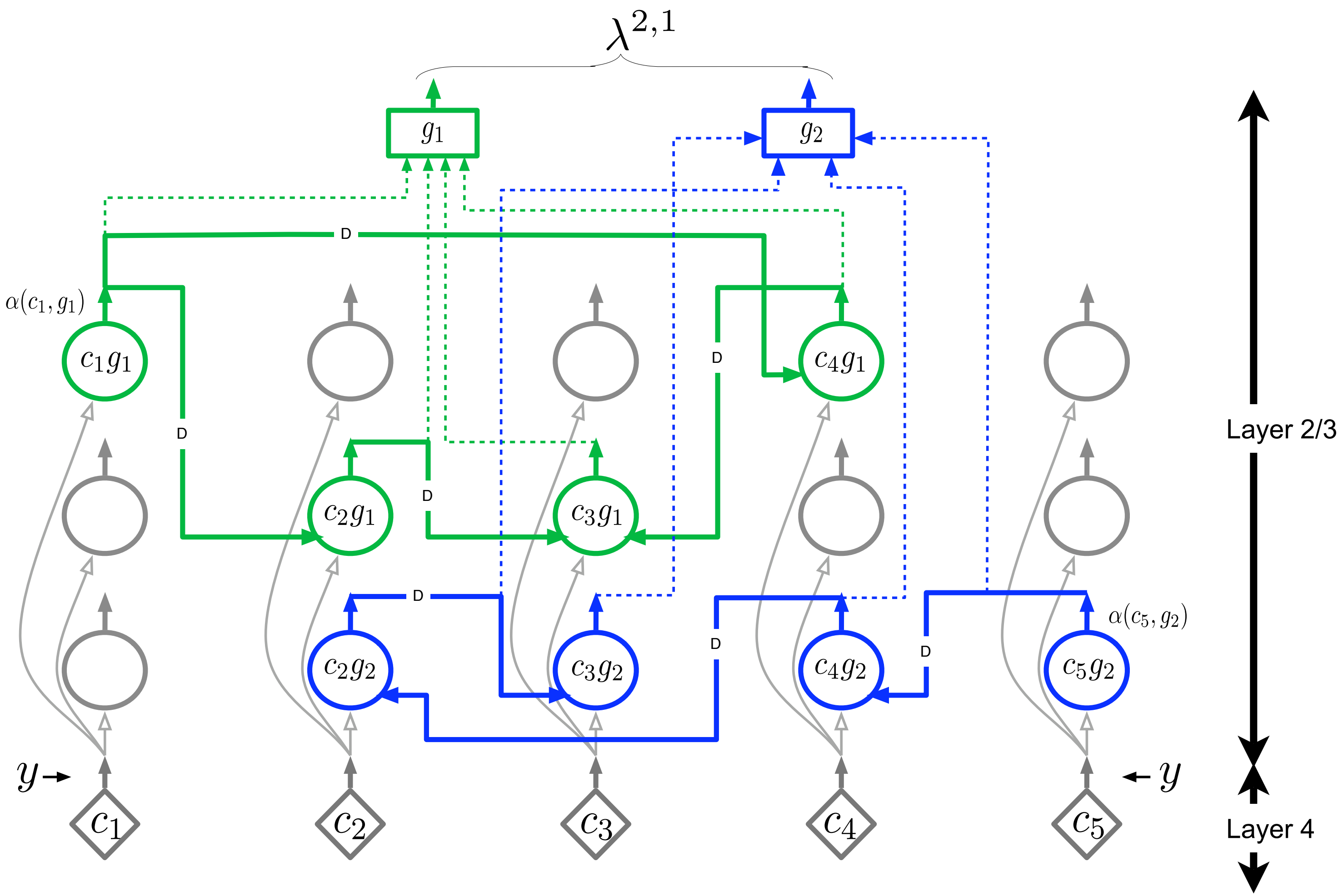

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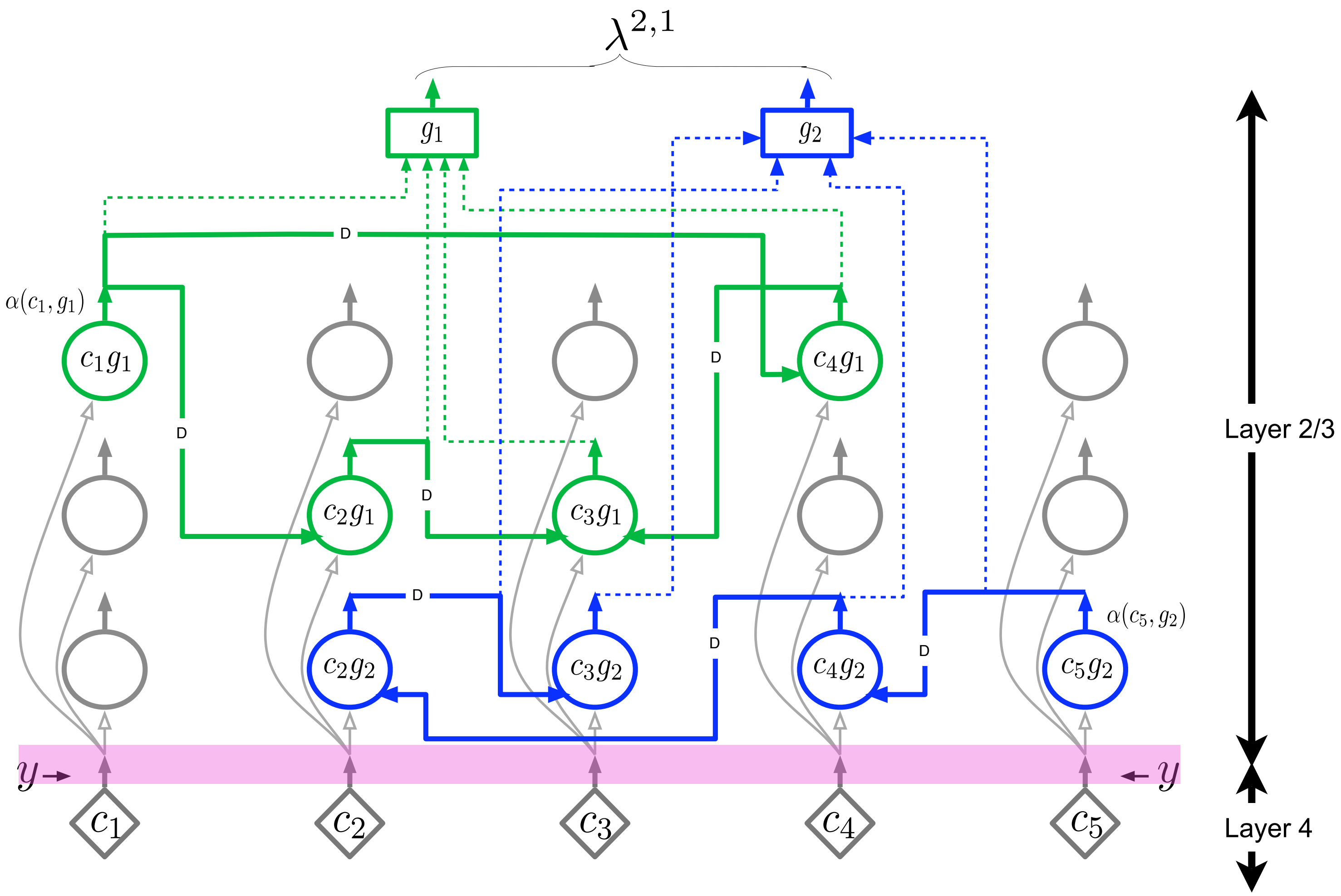
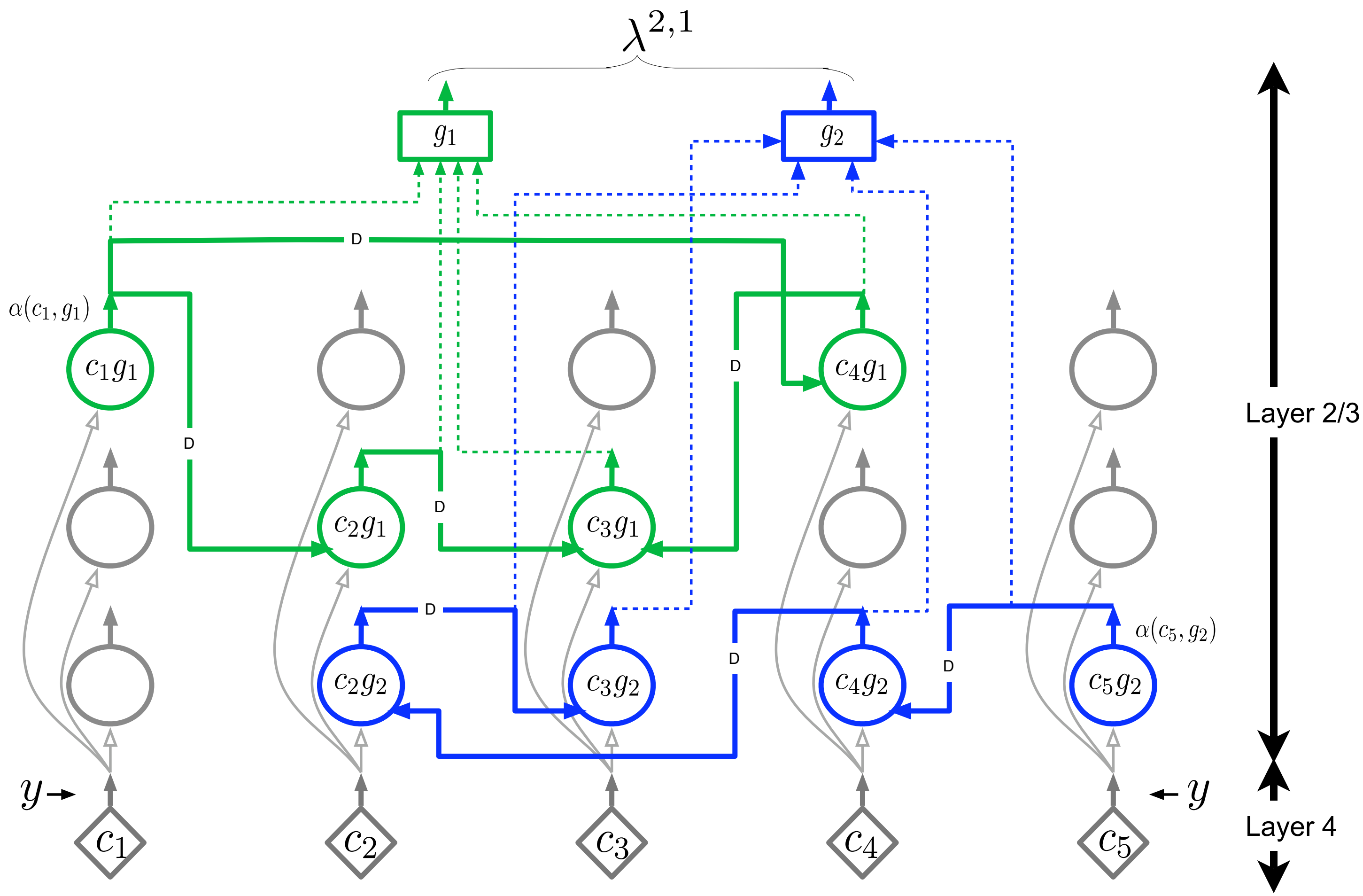


Layer 2/3 circuit (1)



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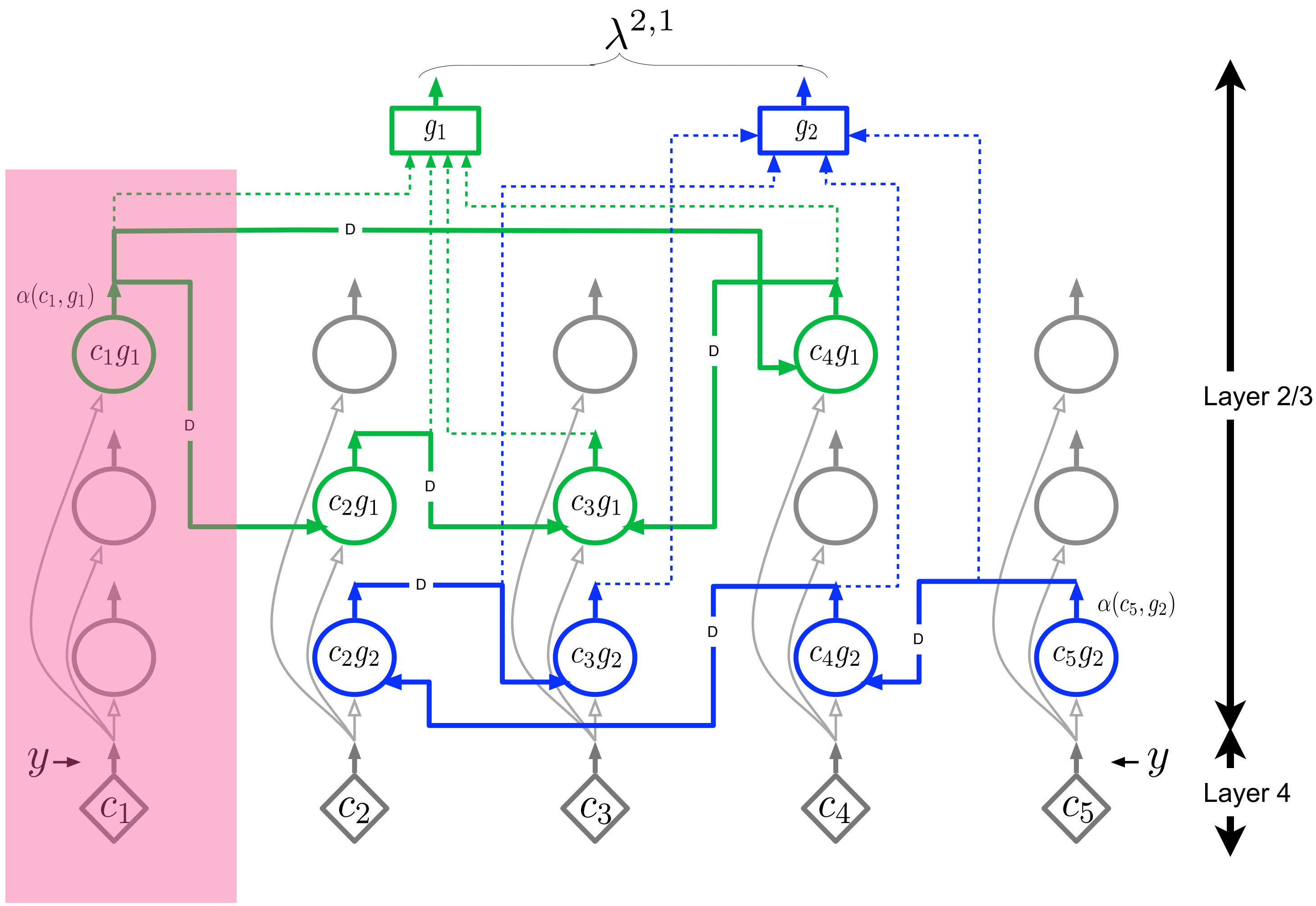
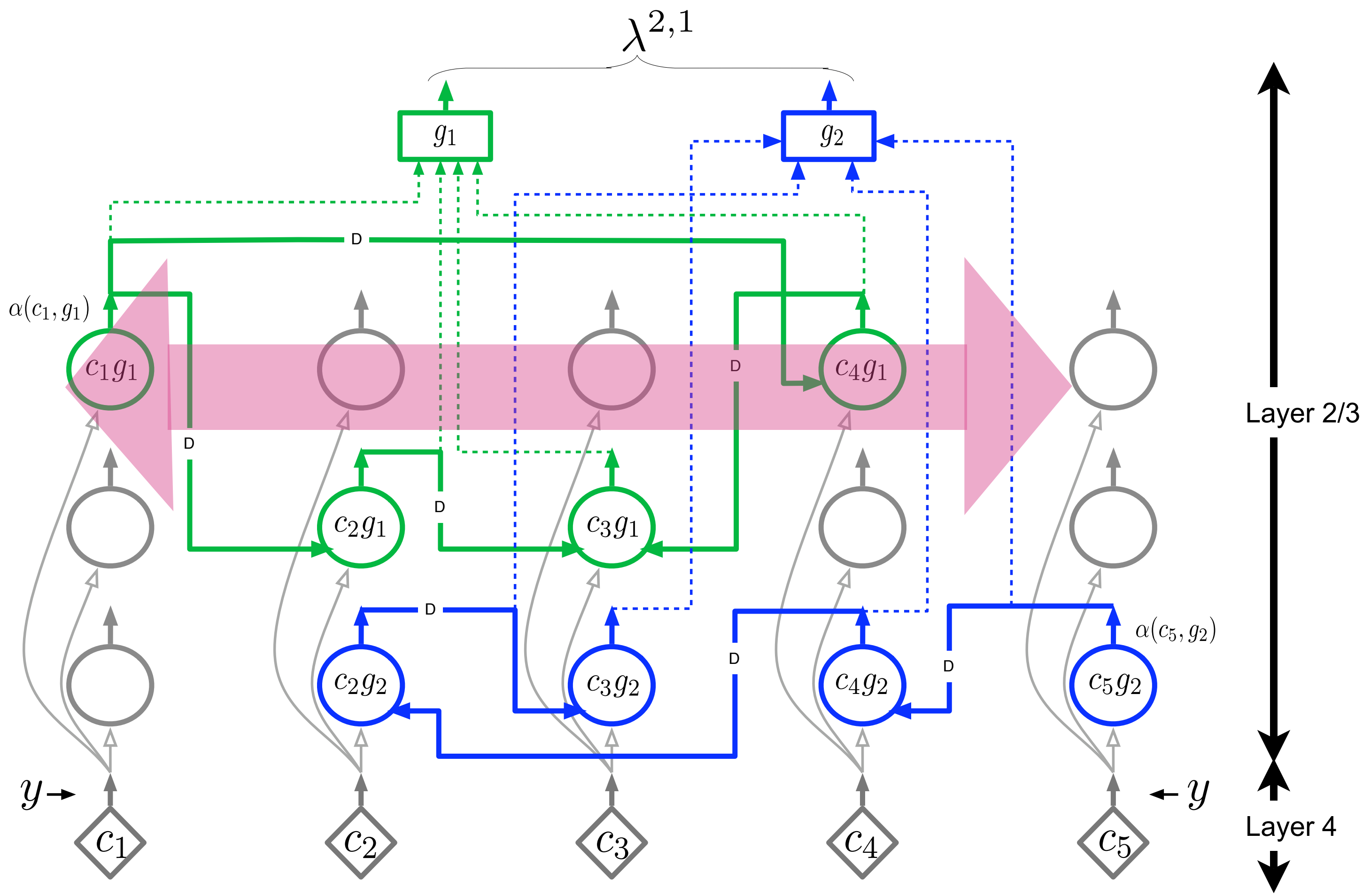


Layer 2/3 circuit (1)



Data : Layer 2/3

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- Major excitatory projection of layer 4 is to layers 2/3.
    - Thomson & Lamy 2007

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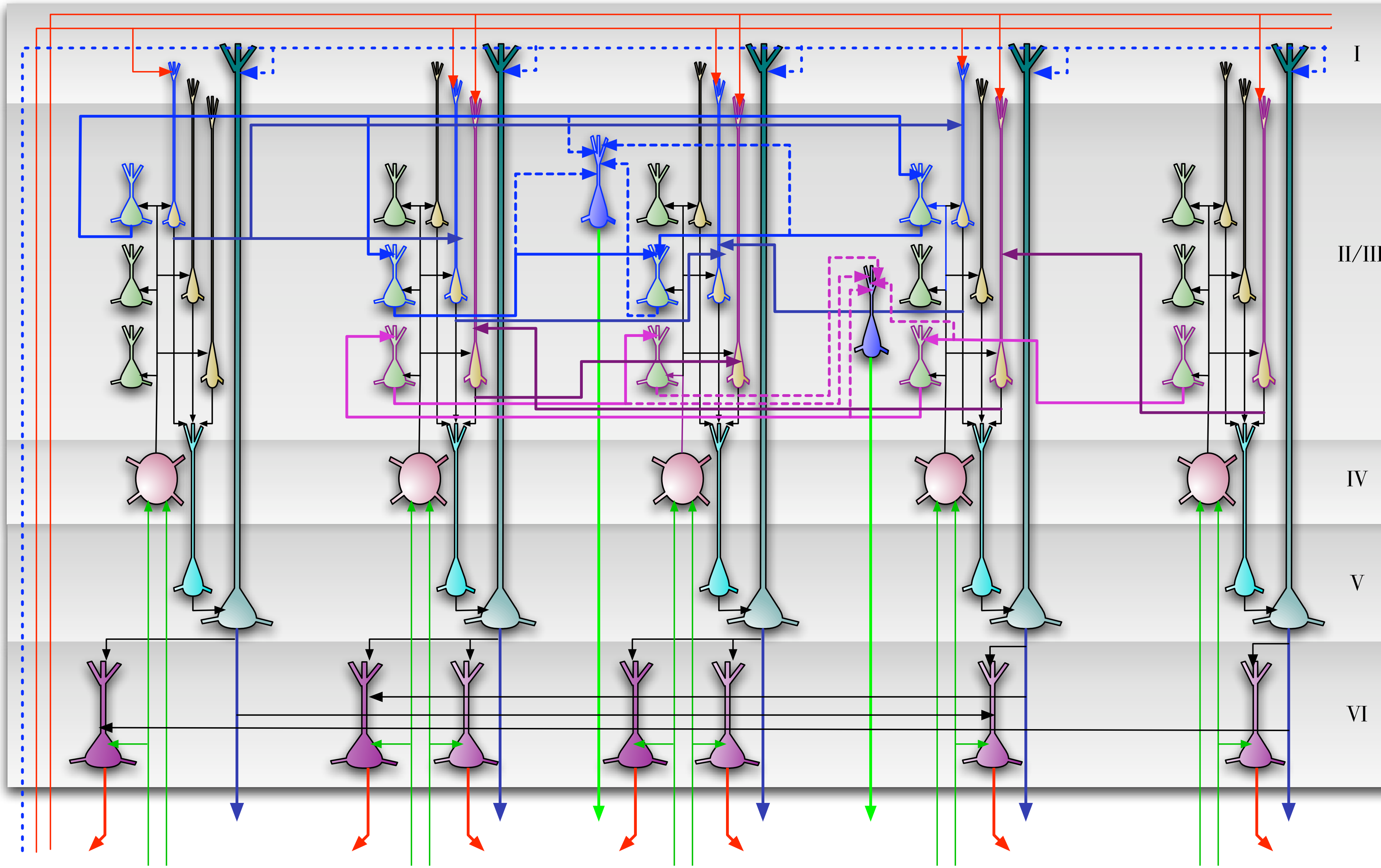
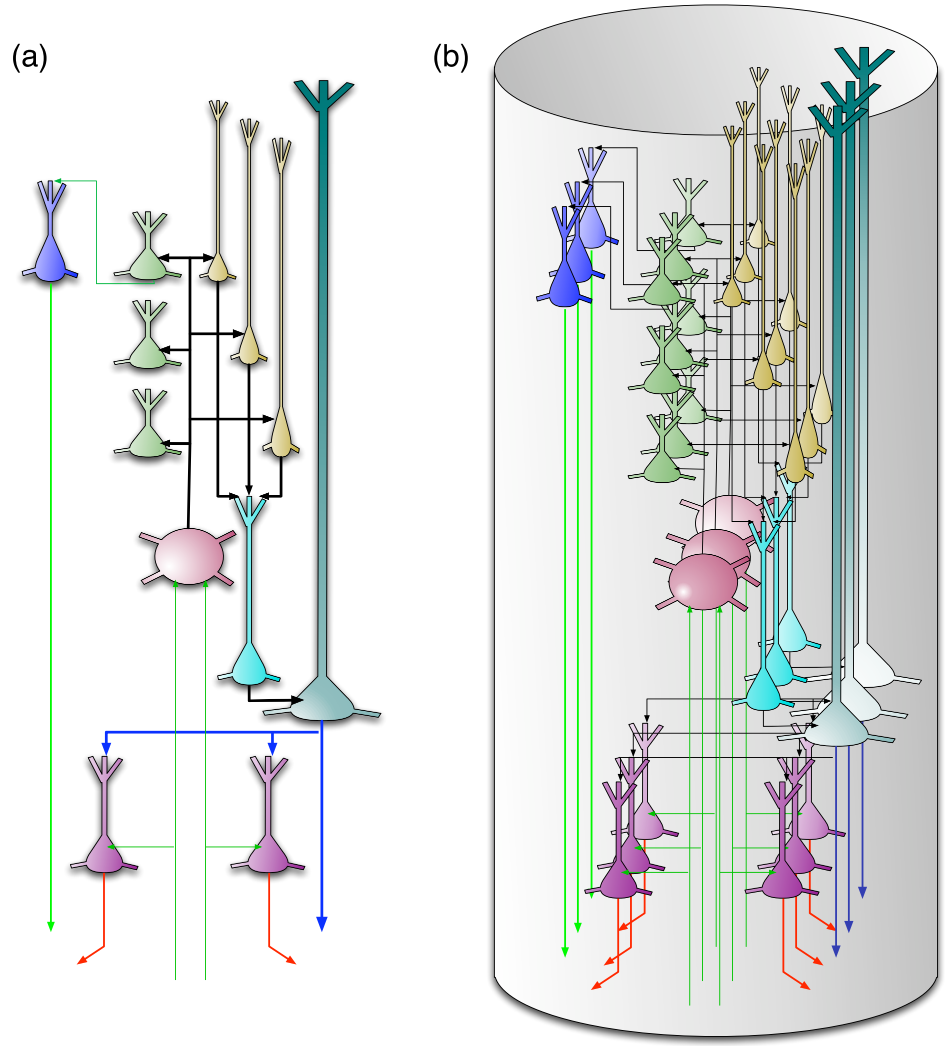
  - Thomson & Lamy 2007
- Layer 4 spiny neurons in the barrel cortex are characterized by vertically oriented intra-columnar axons that target Layer 2/3 pyramidal cells
  - Lubke & Feldmeyer 2007
- Cells in layer 2/3 are complex cells that prefer richer stimuli, such as motion in the preferred direction
  - Hirsch and Martinez 2006

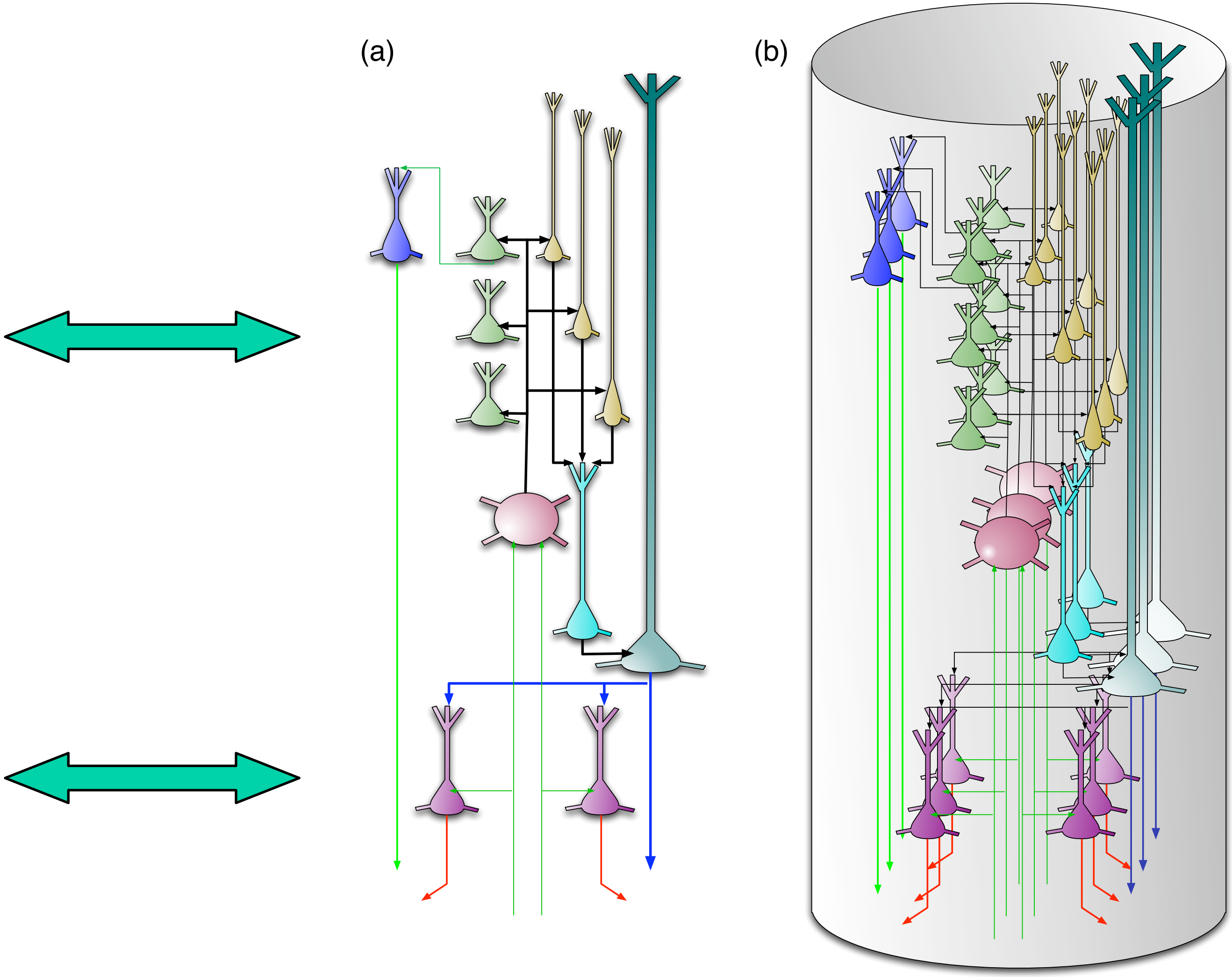
Cortical circuit from HTM belief propagation



Cortical Column



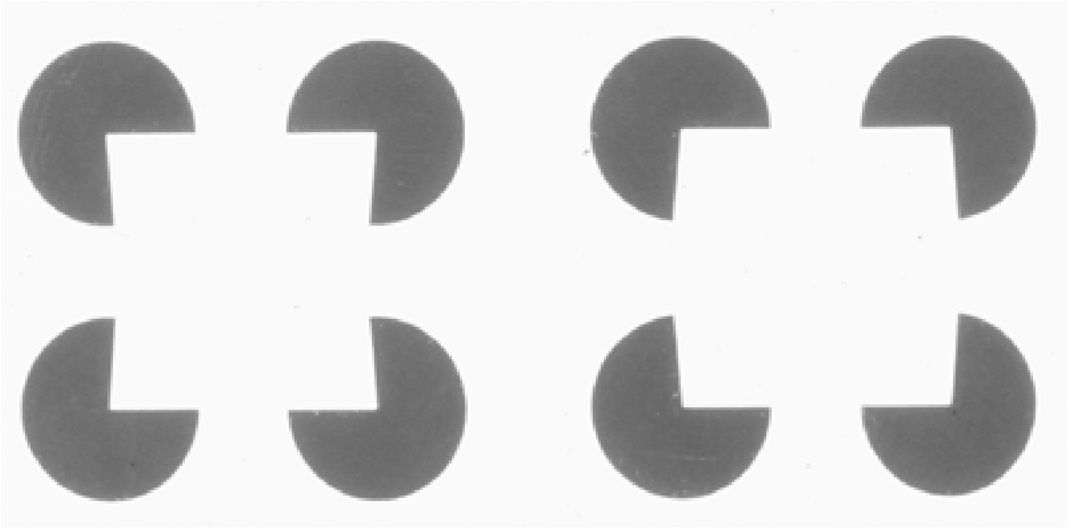
Cortical Column



Summary of laminar functions

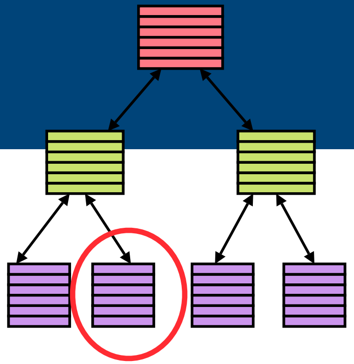
- Layer 4
  - Coincidence detection
- Layer 2/3
  - Feedforward inference on Markov chains (complex cells)
  - Feedback inference on Markov chains (complex cells)
- Layer 5
  - Assimilation (marginalization) of feedback information
  - Timing
- Layer 6
  - Computation of top-down messages from layer 5 outputs

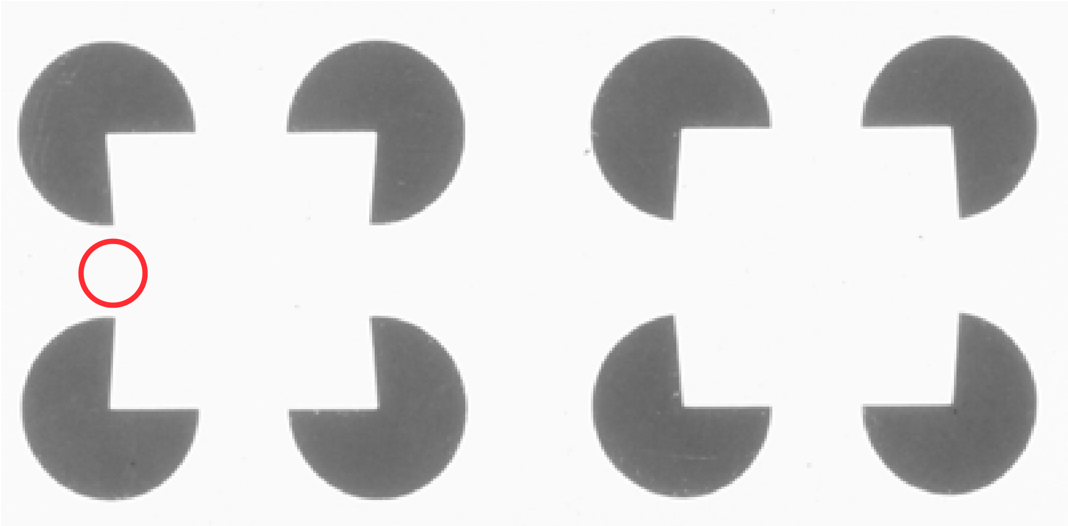
Illusory Contours

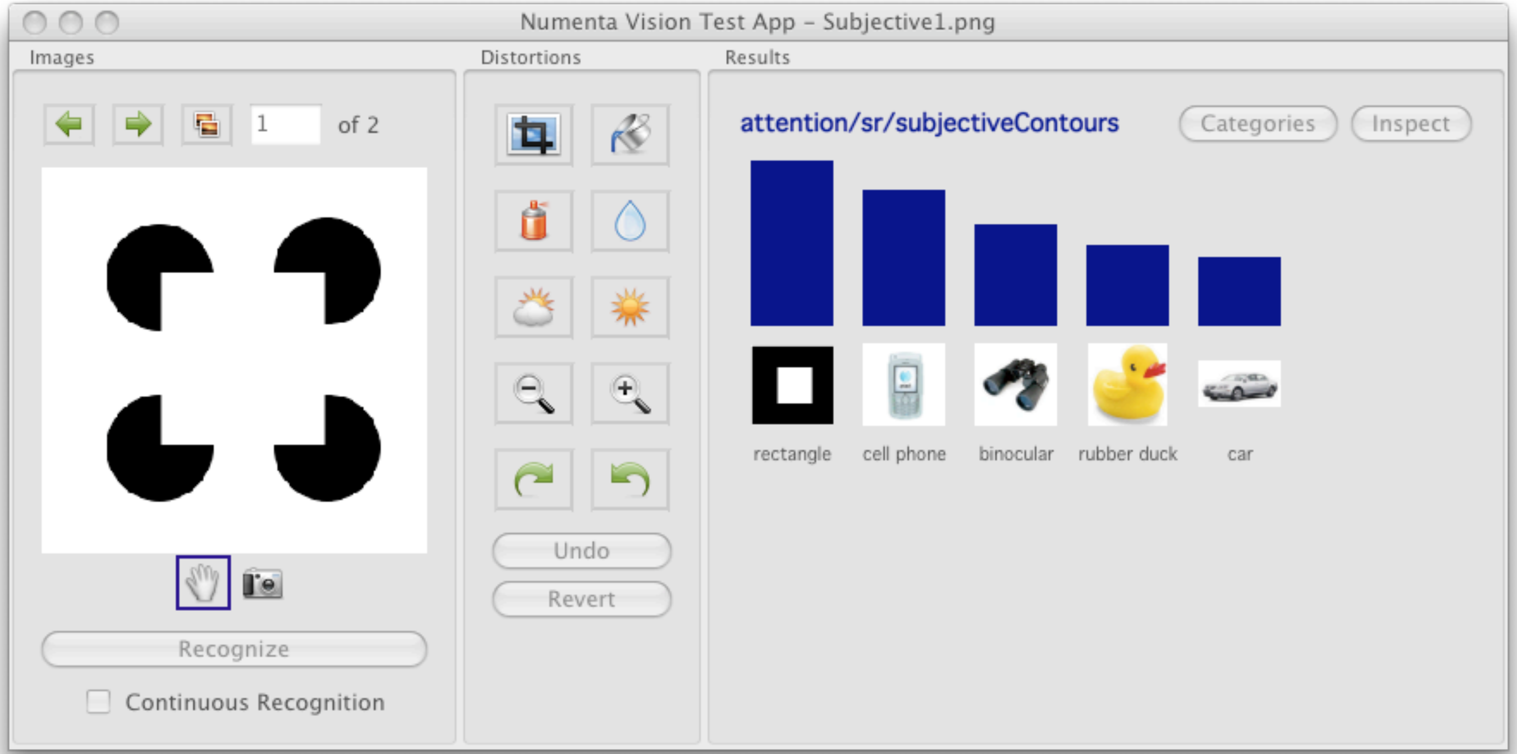


Lee and Mumford, J.Opt. Soc. America. July 2003

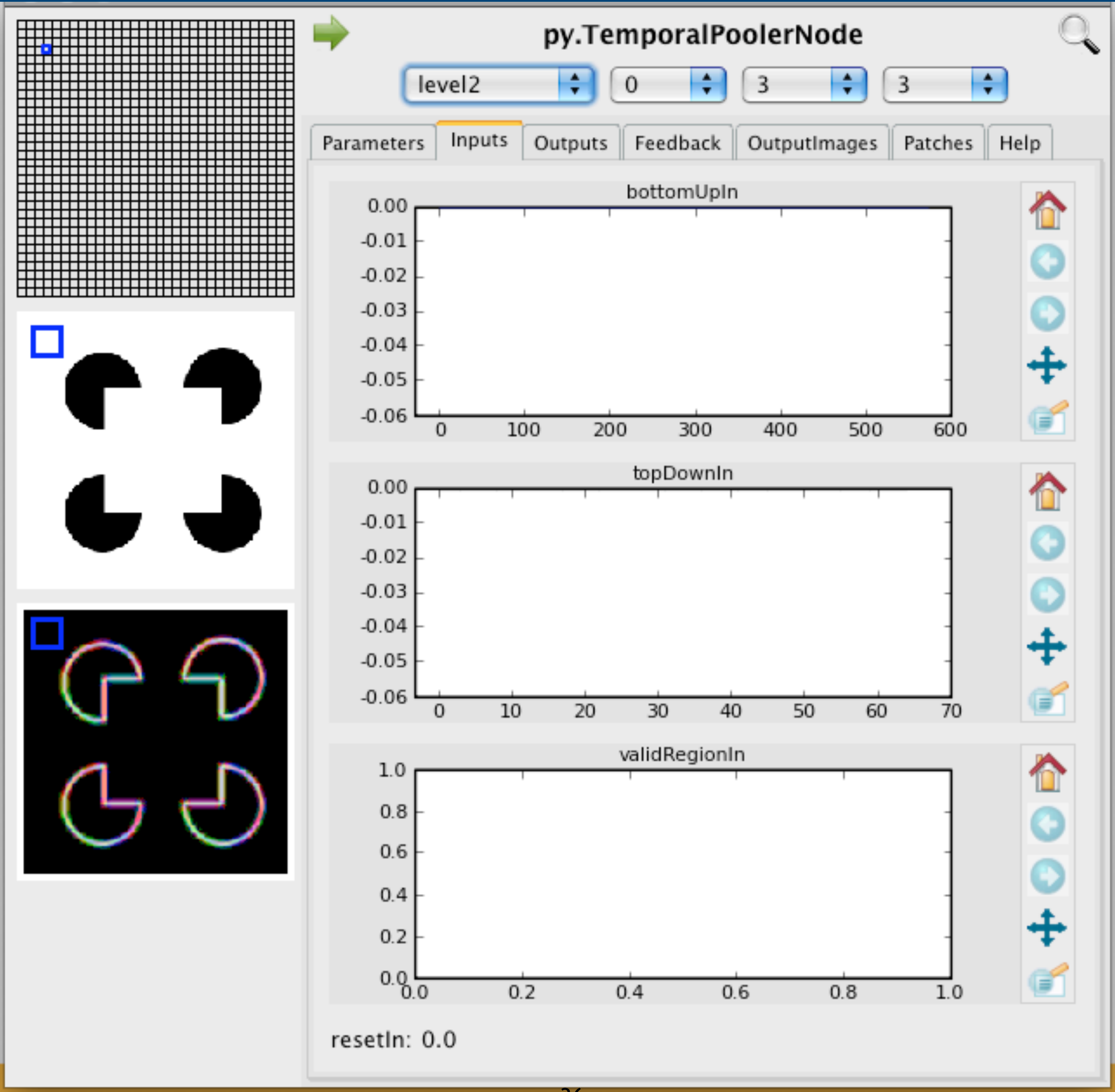
Illusory Contours



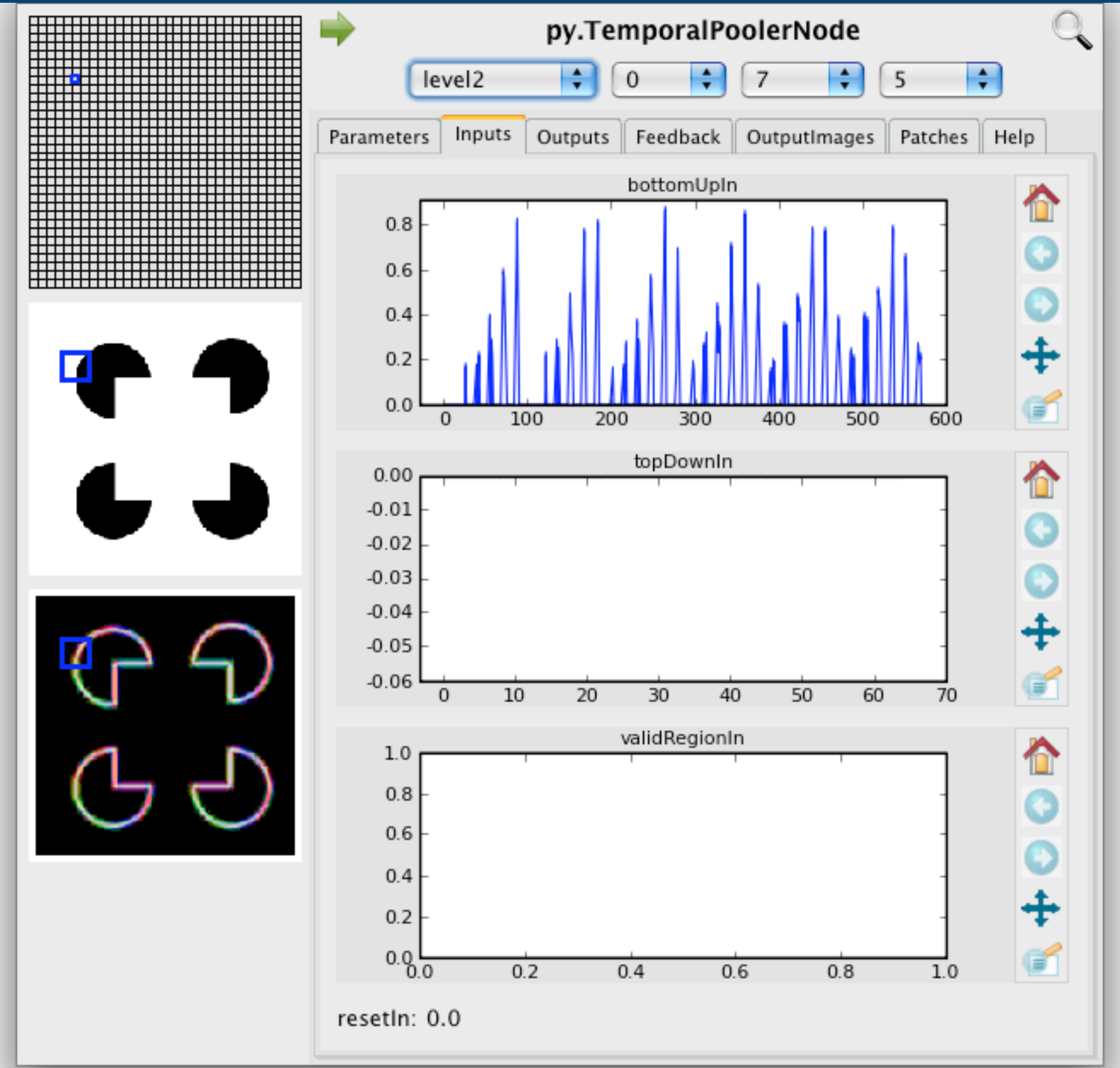



Numenta Vision Test App – Subjective1.png
Images
Distortions
Results
1
of 2
attention/sr/subjectiveContours
Categories
Inspect
rectangle
cell phone
binocular
rubber duck
car
Undo
Revert
Recognize
Continuous Recognition

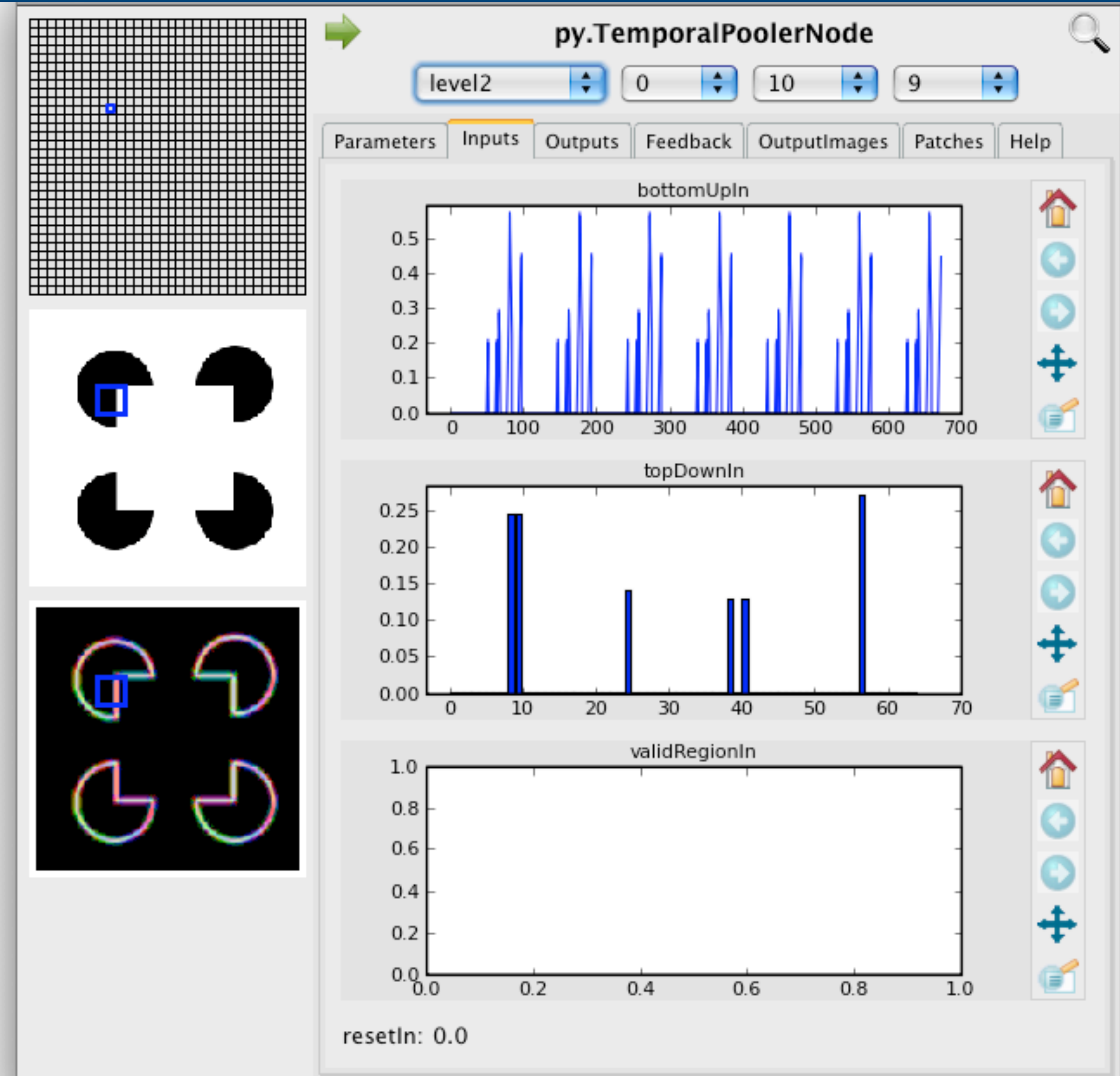
No contours


py.TemporalPoolerNode
level2
0
3
3
Parameters
Inputs
Outputs
Feedback
OutputImages
Patches
Help
bottomUpIn
0.00
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0 100 200 300 400 500 600
topDownIn
0.00
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0 10 20 30 40 50 60 70
validRegionIn
1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.2 0.4 0.6 0.8 1.0
resetIn: 0.0

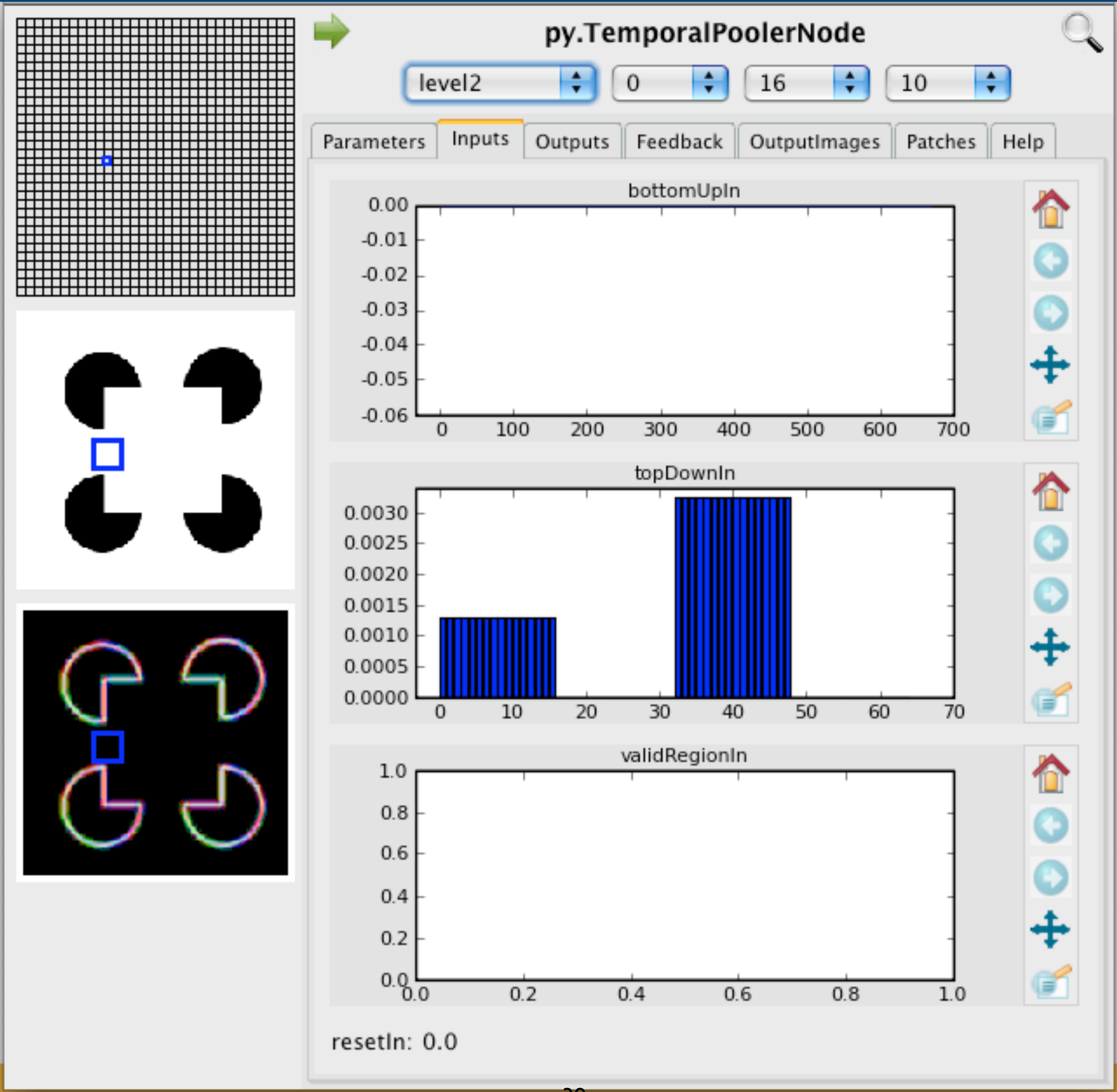
Bottom-up. No Top-down



Bottom-up And Top-Down



Subjective Contour



Computing platform prognosis

- Algorithms are mature enough in application areas like vision
- It is very likely that there will be embedded applications in near future
- The first embedded circuits are likely to be inference-only
  - Learning done offline using parallel software
- Time is right to start specing out a computational platform for HTM-like computation

To learn more

- Download Vision4 Demo from www.numenta.com
- Read On Intelligence
- Read white papers from Numenta website
- Read Dileep's PhD thesis *"How the brain might work...*
  - search for "Dileep thesis" on google

Thank You